Neutron star crust matter

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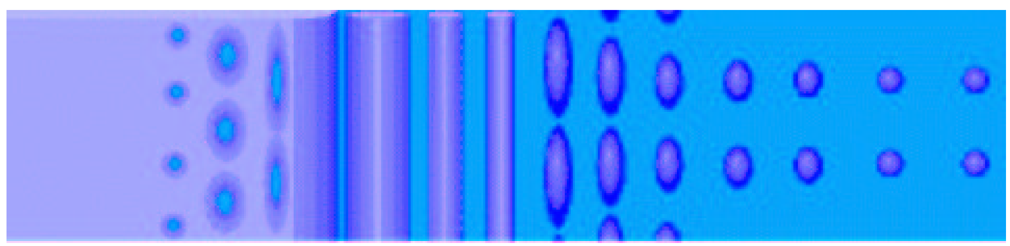
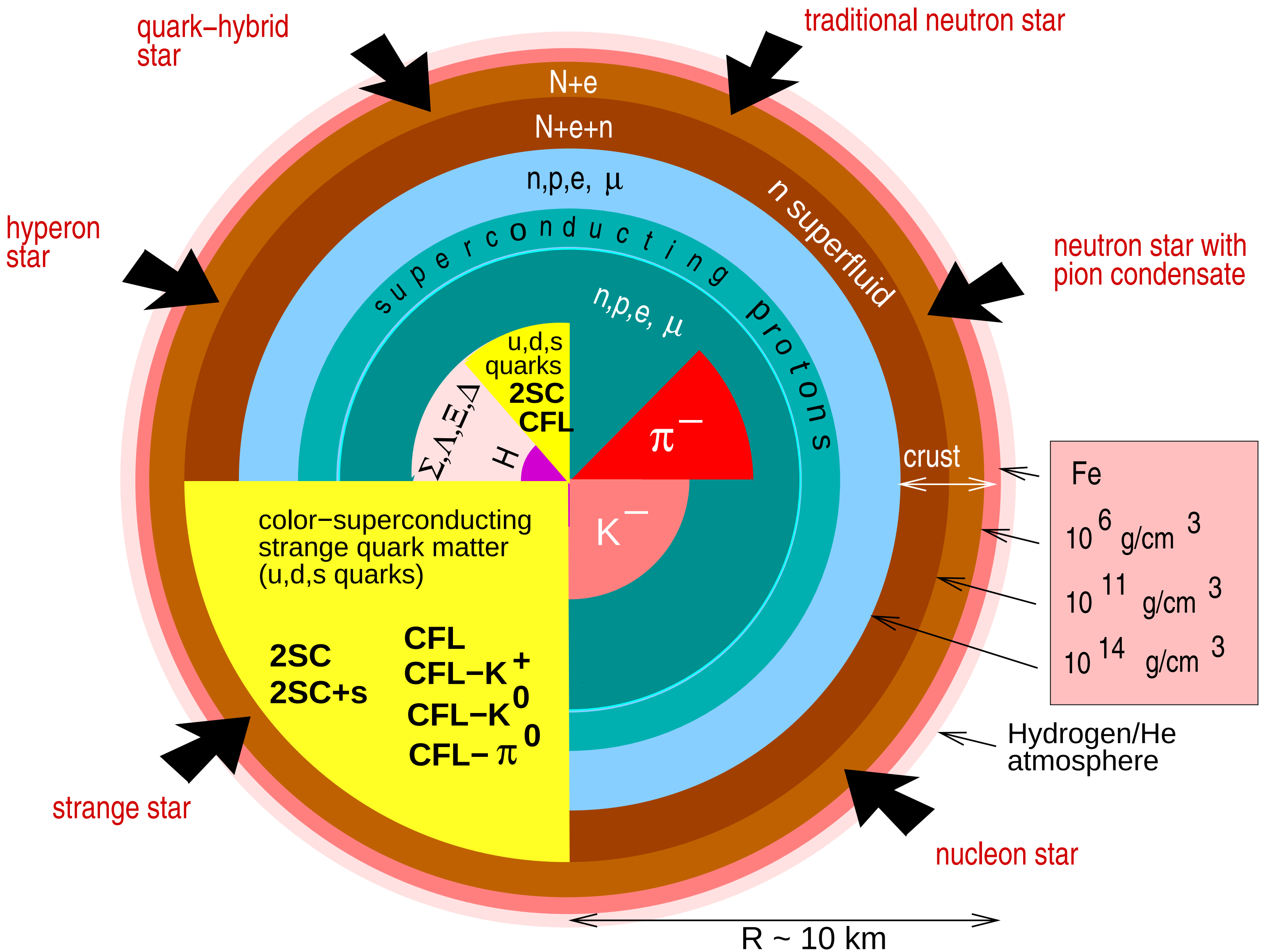


Helmholtz international Summer School, July 26-August 4, Dubna

Guidelines

- ★ Introduction
- ★ Structure of neutron star crust
  - ⋆ Overview of calculations
  - ⋆ Negele& Vautherin
- ★ Superfluidity in the crust
  - ⋆ Pairing field in the presence of nuclei
  - ⋆ Effects of pairing on the structure of the crust
  - ⋆ Observational contraints
- ★ Transport properties
  - ⋆ From solid state to nuclear physics
  - ⋆ Bragg scattering and effective mass
- ★ Summary & perspectives
- ★ Bibliography

Neutron stars



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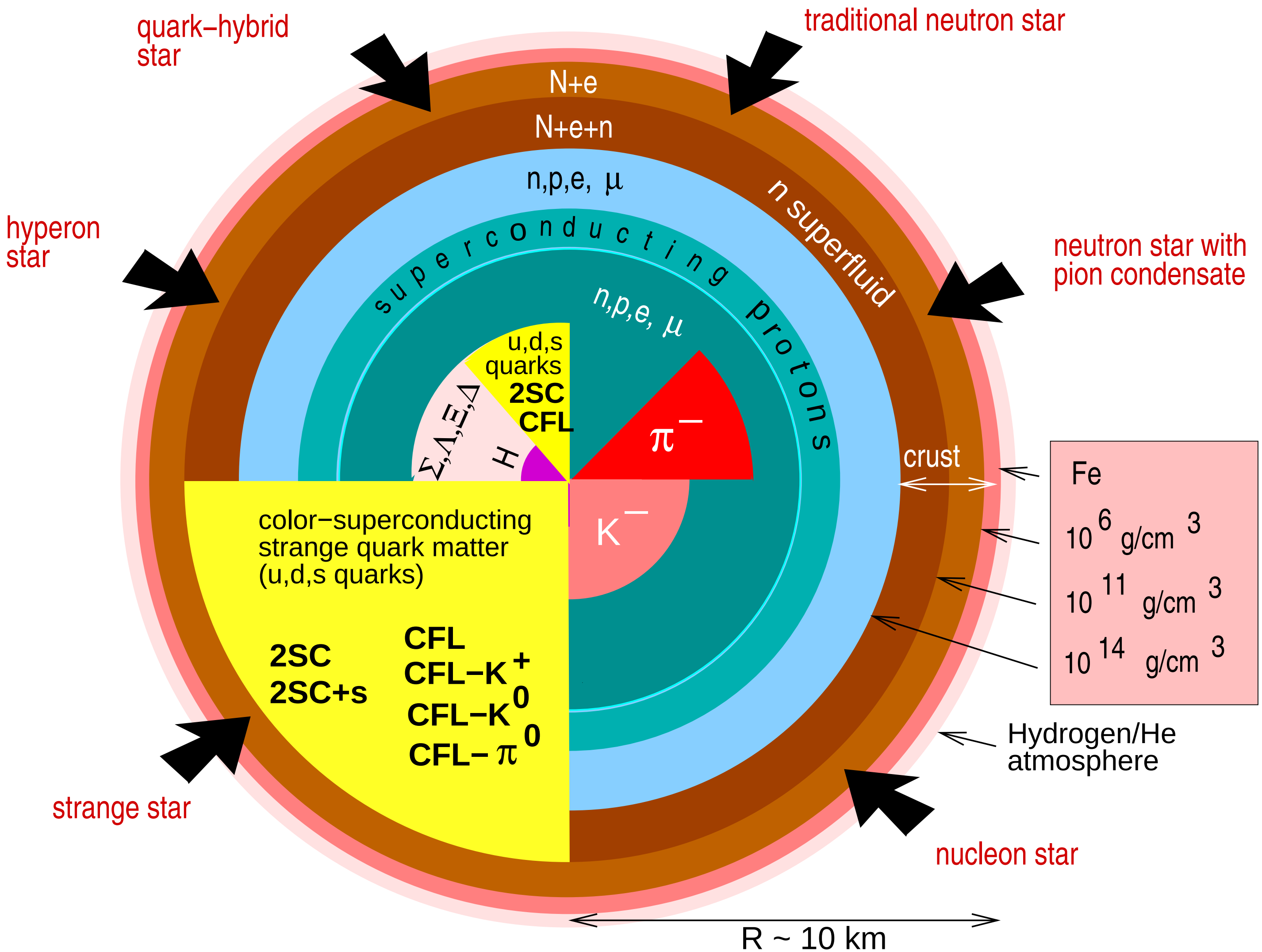

Neutron star crust $\sim$ 1% mass, 10% radius

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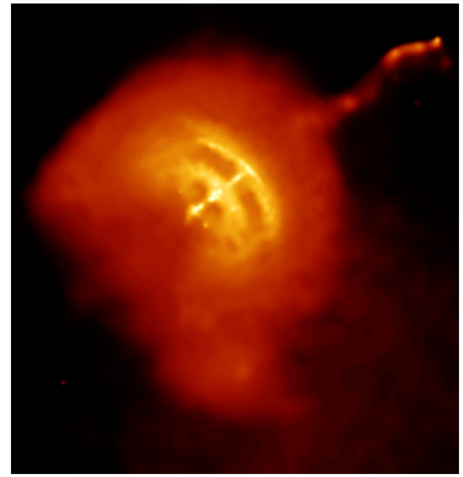
Crust=interface between outer layers (observations) and core



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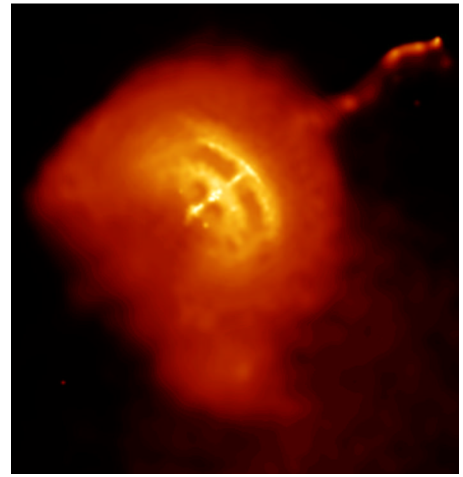

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 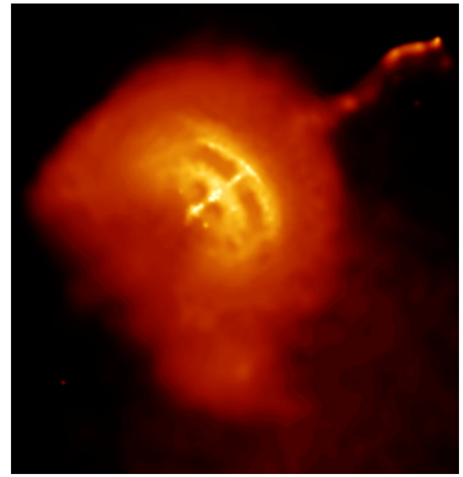

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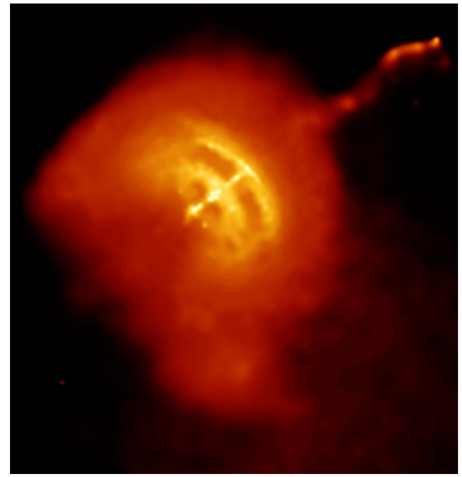

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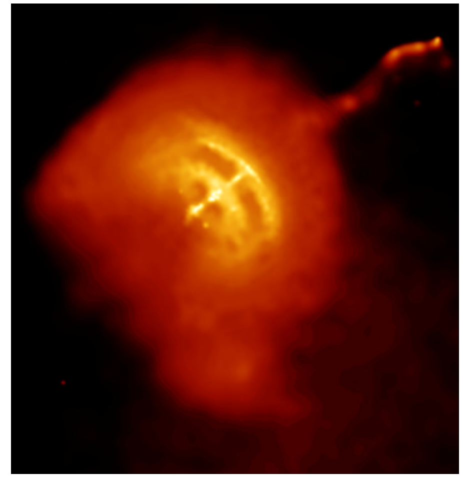

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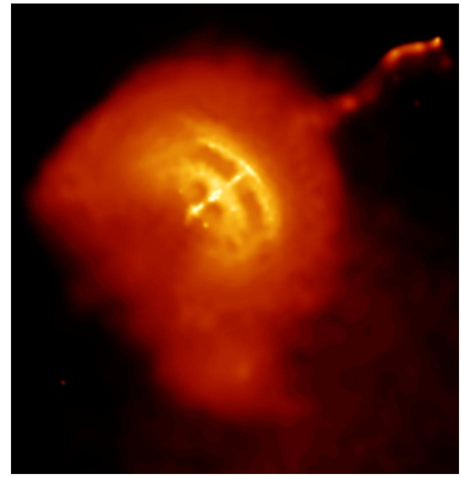

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★ elastic properties ⇒ pulsar glitches, oscillation modes, gravitational waves

★ equation of state ⇒ binary merger

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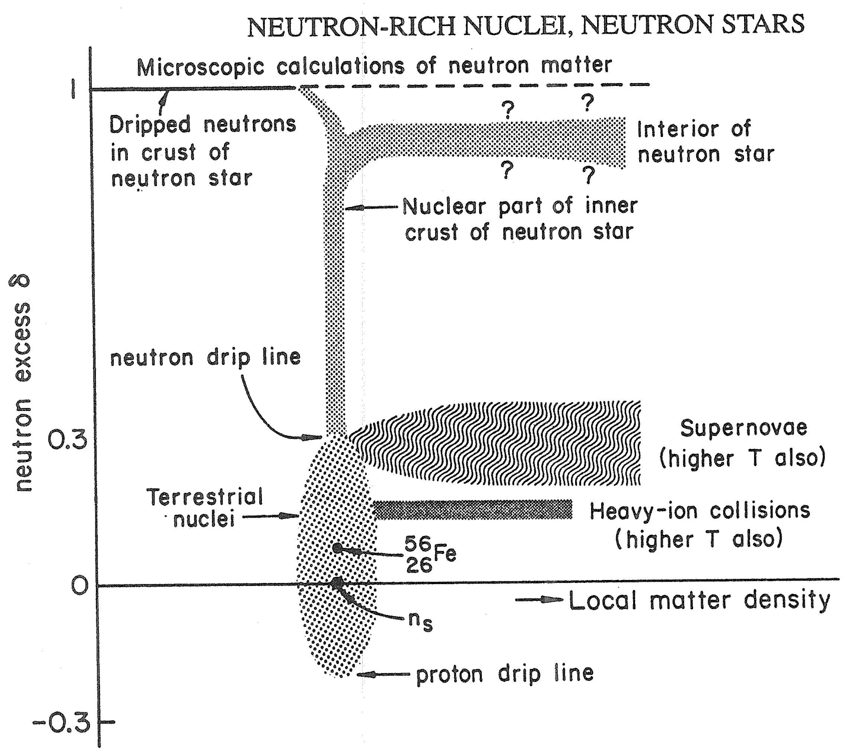

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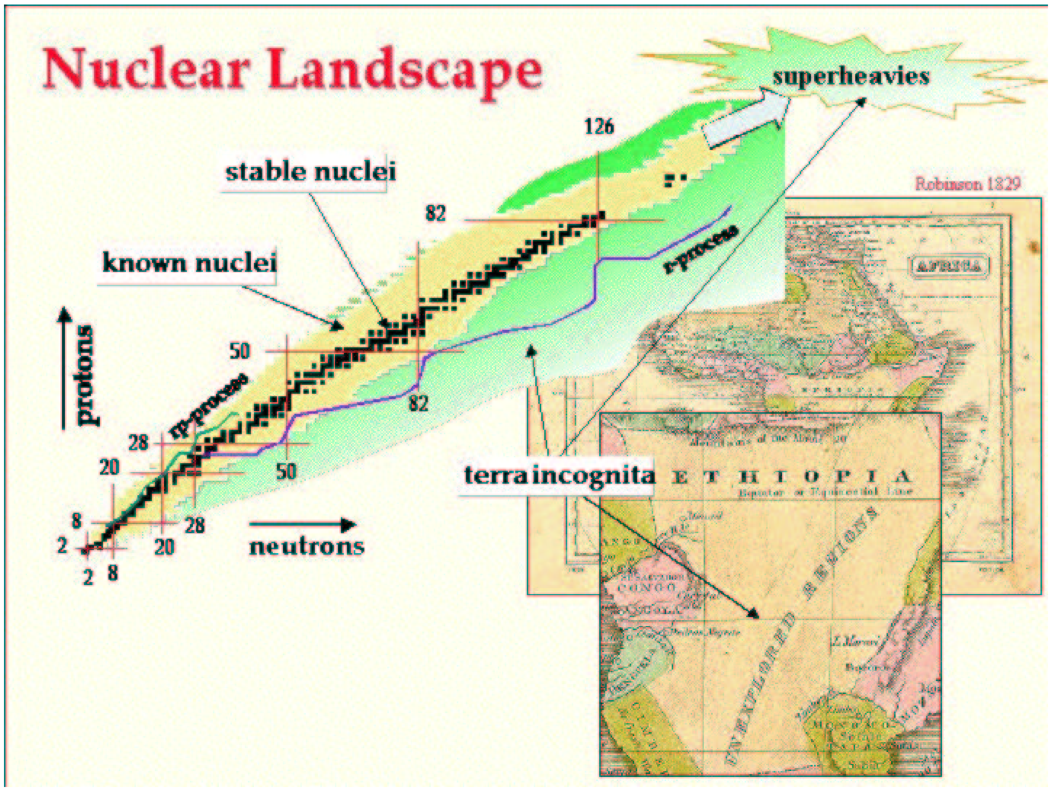

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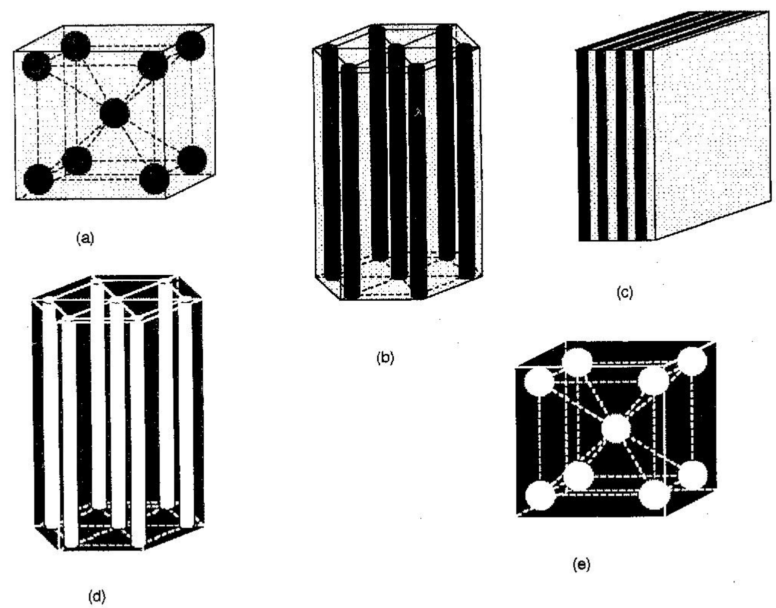

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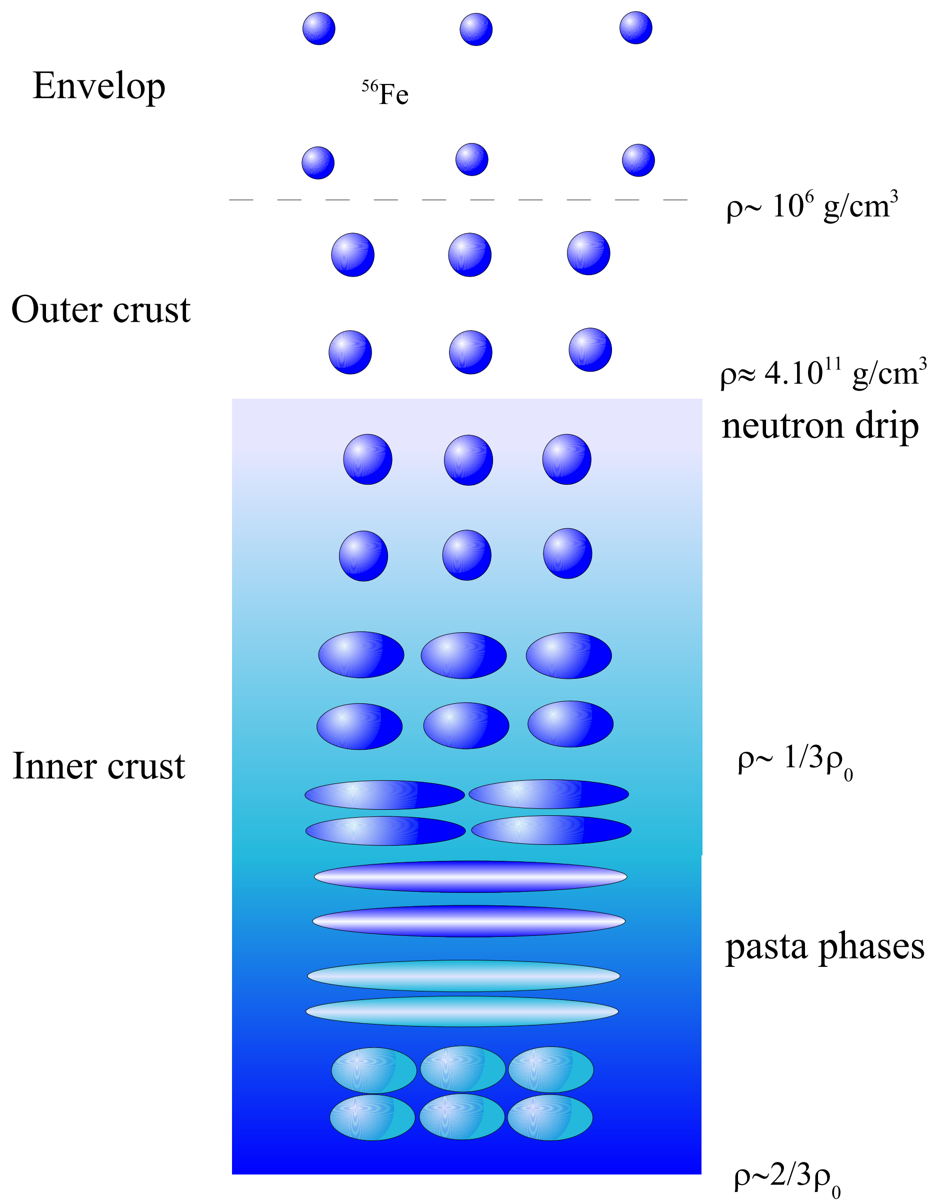
- ★ Strongly deformed nuclei (“pasta” phases)
- ★ Nuclei immersed in a neutron superfluid

$\Rightarrow$ nuclear astrophysical laboratory !

Structure of the crust



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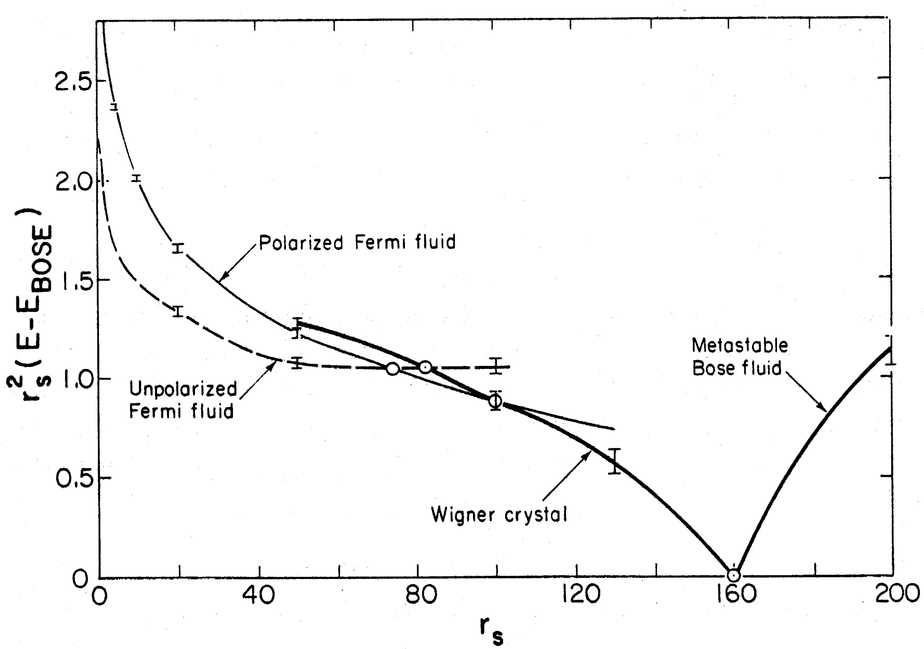

$r_s \equiv a/a_0,\ a_0 = \hbar^2/m_e e^2$
in metals $r_s \sim 2-6$
in NS crust $r_s \sim 10^{-5} - 10^{-2}$

Ceperley *et al.*, PRL 45 (1980) 569.

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$$\varepsilon = n_N E\{A, Z\} + \varepsilon_e + \varepsilon_L$$

$E\{A, Z\}$ energy of a nucleus (mass of known nuclei or semi-empirical mass formula)
ε_e energy density of the electron gas
ε_L lattice energy density

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$$\varepsilon = n_N E\{A, Z\} + \varepsilon_e + \varepsilon_L$$

uniform relativistic electron gas

$$\varepsilon_e = \frac{m_e^4 c^5}{8\pi^2\hbar^3}\left(x(\sqrt{1+x^2}(1+2x^2) - \log\{x + \sqrt{1+x^2}\}\right)$$

where $x = \hbar k_{eF}/m_e c$ and $k_{eF} = (3\pi^2 n_e)^{1/3}$

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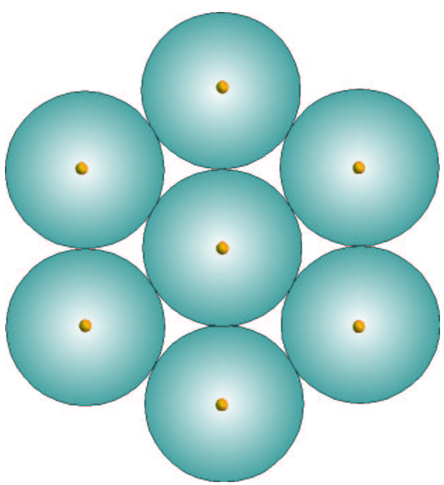
Wigner-Seitz approximation



Each sphere is electrically neutral
$\Rightarrow \varepsilon_L = \varepsilon_{ee} + \varepsilon_{eN}$
assuming uniform electron sea

$$\Rightarrow \varepsilon_L = -\frac{9}{10}\frac{Z^2 e^2}{R_{\rm cell}} n_N \left(1 - \frac{5}{9}\frac{<r^2>}{R_{\rm cell}^2}\right)$$

Composition of the outer crust (T=0)

The structure of the outer crust up to $\rho \sim 10^{11}$ g.cm^{-3} is completely determined by the measured masses of neutron rich nuclei, Haensel & Pichon Astron. & Astrophys. 283 (1994) 313.

Element	Z	N	Z/A	ρ_{max} (g cm^{-3})	μ_e (MeV)	$\Delta\rho/\rho$ (%)
^{56}Fe	26	30	0.4643	$7.96\ 10^{6}$	0.95	2.9
^{62}Ni	28	34	0.4516	$2.71\ 10^{8}$	2.61	3.1
^{64}Ni	28	36	0.4375	$1.30\ 10^{9}$	4.31	3.1
^{66}Ni	28	38	0.4242	$1.48\ 10^{9}$	4.45	2.0
^{86}Kr	36	50	0.4186	$3.12\ 10^{9}$	5.66	3.3
^{84}Se	34	50	0.4048	$1.10\ 10^{10}$	8.49	3.6
^{82}Ge	32	50	0.3902	$2.80\ 10^{10}$	11.44	3.9
^{80}Zn	30	50	0.3750	$5.44\ 10^{10}$	14.08	4.3
^{78}Ni	28	50	0.3590	$9.64\ 10^{10}$	16.78	4.0
^{126}Ru	44	82	0.3492	$1.29\ 10^{11}$	18.34	3.0
^{124}Mo	42	82	0.3387	$1.88\ 10^{11}$	20.56	3.2
^{122}Zr	40	82	0.3279	$2.67\ 10^{11}$	22.86	3.4
^{120}Sr	38	82	0.3167	$3.79\ 10^{11}$	25.38	3.6
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$\Rightarrow$ strong shell effects with magic numbers 28, 50, 82

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From a given "microscopic" Hamiltonian :

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- Hartree-Fock-Bogoliubov
  - independent quasiparticles $\Rightarrow$ pairing effects

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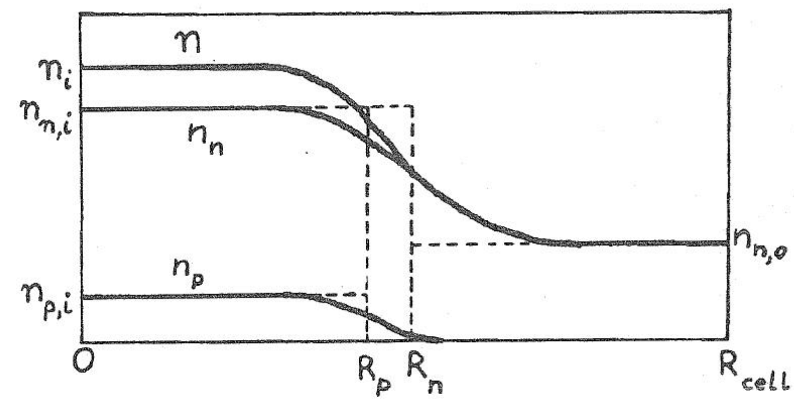
Compressible liquid drop model
(+W-S approximation)

$\varepsilon = \varepsilon_{N,\mathrm{bulk}} + \varepsilon_{N,\mathrm{surf}} + \varepsilon_C + \varepsilon_e$



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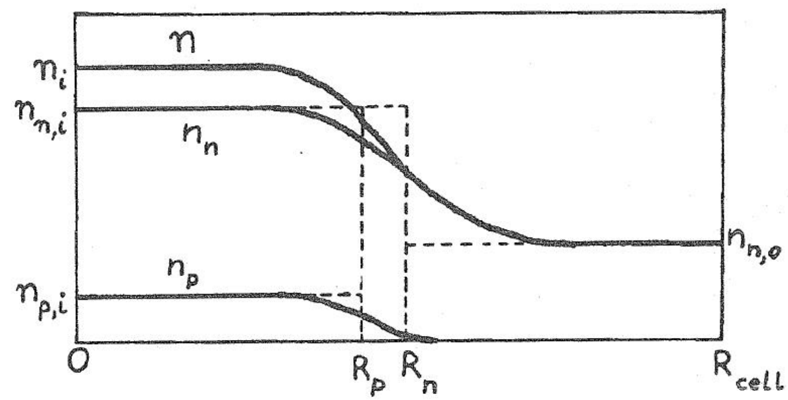

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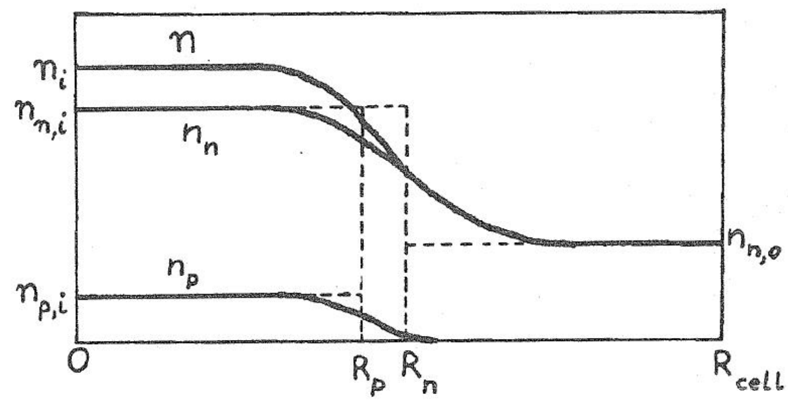

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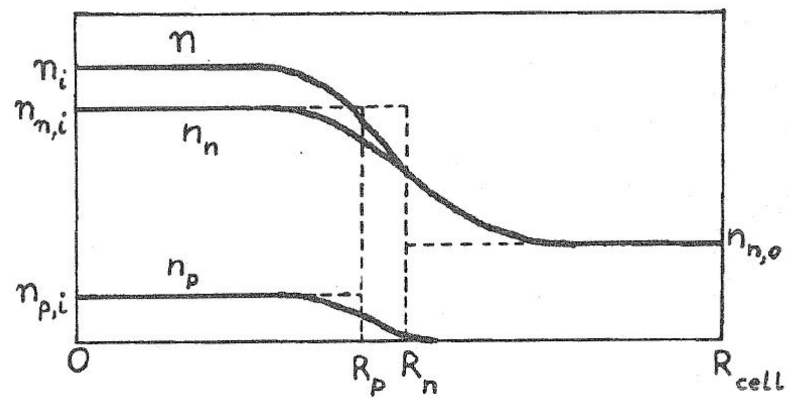

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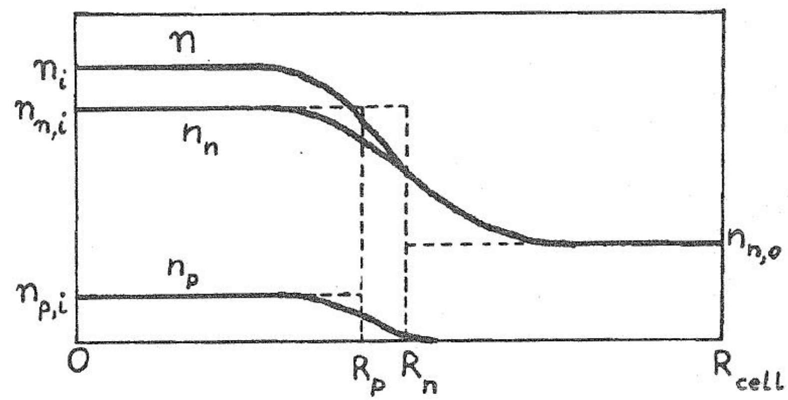

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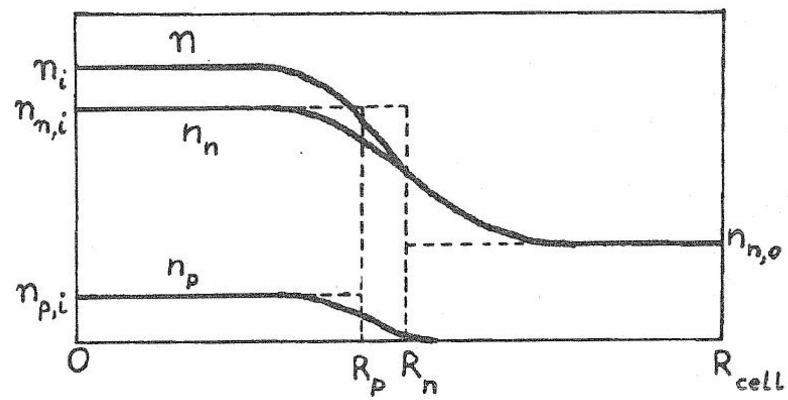

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$$u = (R_{\mathrm{p}}/R_{\mathrm{cell}})^3$$
$W\{n_{\mathrm{n}}, n_{\mathrm{p}}\}$ energy density of homogeneous nuclear matter
$$\varepsilon_{N,\mathrm{surf}} = (\mathcal{A}\sigma + N_s \mu_{\mathrm{n},s})/\mathcal{V}_{\mathrm{cell}}$$

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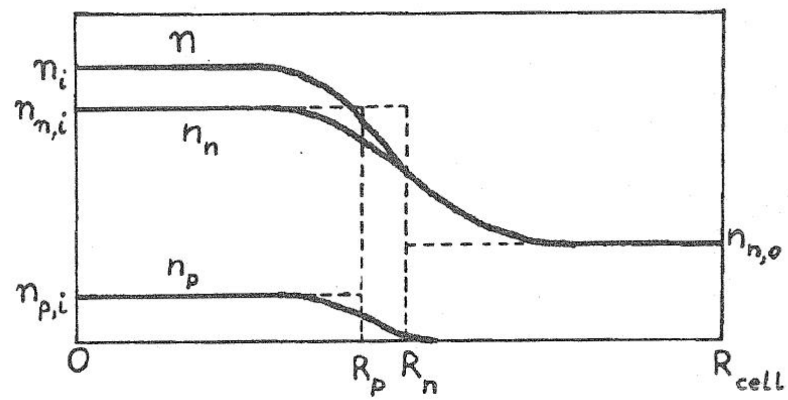

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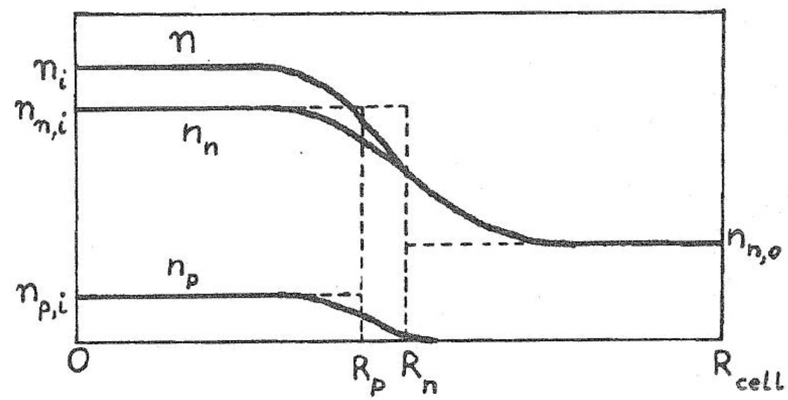
$$\varepsilon_C = \varepsilon_{N,C} + \varepsilon_L = \frac{4\pi}{5}(n_{\mathrm{p},i}e)^2 u {R_\mathrm{p}}^2 \left(1 - \frac{3}{2}u^{1/3} + \frac{1}{2}u\right)$$

Ground state of matter above neutron drip

Compressible liquid drop model
(+W-S approximation)
$\varepsilon = \varepsilon_{N,\text{bulk}} + \varepsilon_{N,\text{surf}} + \varepsilon_C + \varepsilon_e$



$$\varepsilon_{N,\text{bulk}} = uW\{n_{\text{n},i}, n_{\text{p},i}\} + (1-u)W\{n_{\text{n},o}, 0\}$$

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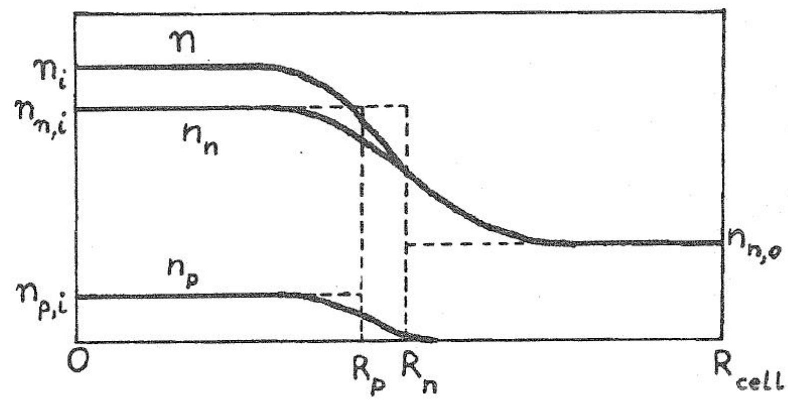

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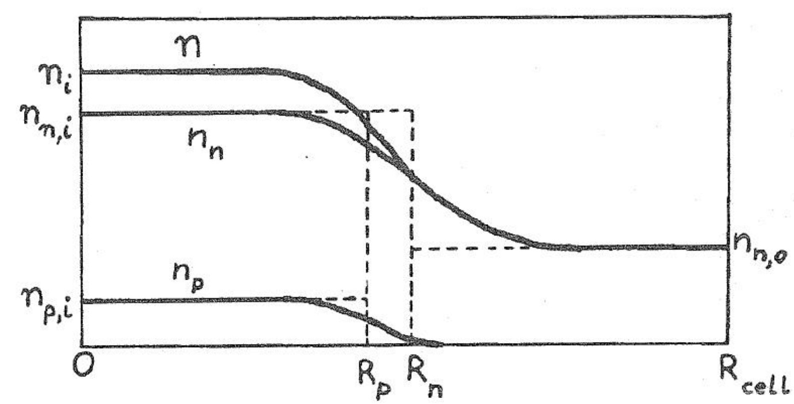

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Consider a 2 phase nucleon system in thermodynamical equilibrium at $T = 0$ separated by a plane interface at $z = z_{\mathrm{ref}}$.

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$$\varepsilon_{{\rm bulk},z_{\rm ref}}\{z\} = \varepsilon_i\, \theta\{z_{\rm ref} - z\} + \varepsilon_o\, \theta\{z - z_{\rm ref}\}$$

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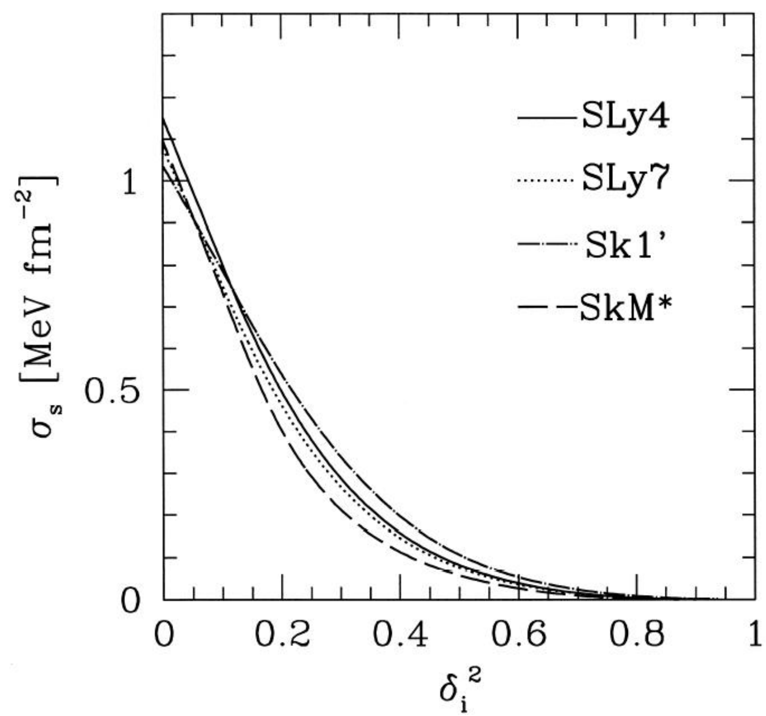
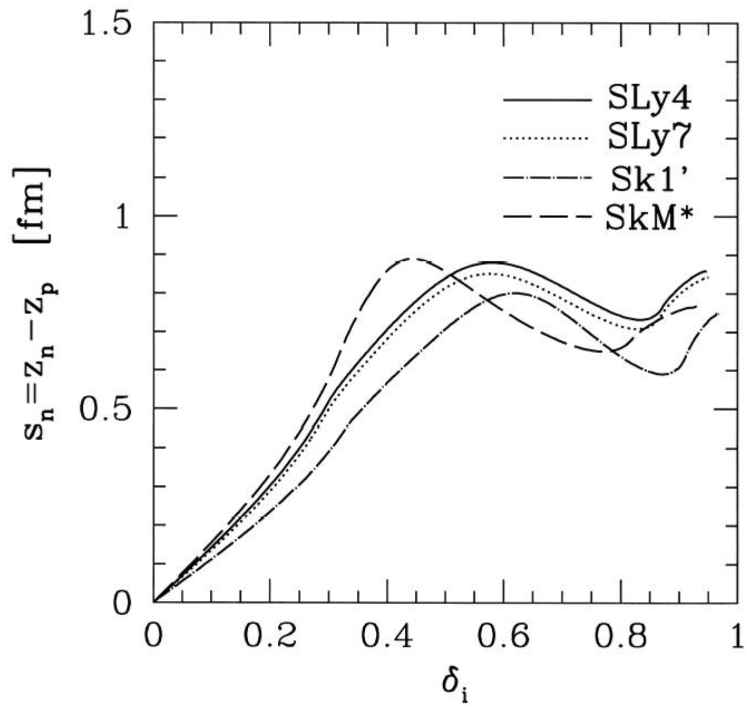
Surface tension

Douchin *et al.*, Nucl. Phys. A 665 (2000) 419-446.
ε calculated from Skyrme energy density functional within ETF approximation





Virial theorem

Baym *et al.*, Nucl. Phys. A175 (1971) 225.

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$\Rightarrow$ At equilibrium

$$\boxed{\varepsilon_{N,surf} = 2\varepsilon_C}$$

$\Rightarrow$ The lattice energy plays a crucial role for determining the composition and the shape of nuclei !

$\varepsilon_L \sim 15\%$ of the total Coulomb energy at $\rho \sim 10^{11}$ g.cm^{-3}

Equilibrium conditions

Equilibrium conditions

★ Mechanical equilibrium ⇒ generalised Laplace's formula

$$P_i^{\text{bulk}} - P_o^{\text{bulk}} = \frac{2\sigma_s}{R_{\text{p}}} - \frac{4\pi}{15} e^2 n_{\text{p},i}^2 R_{\text{p}}{}^2 (1-u)$$

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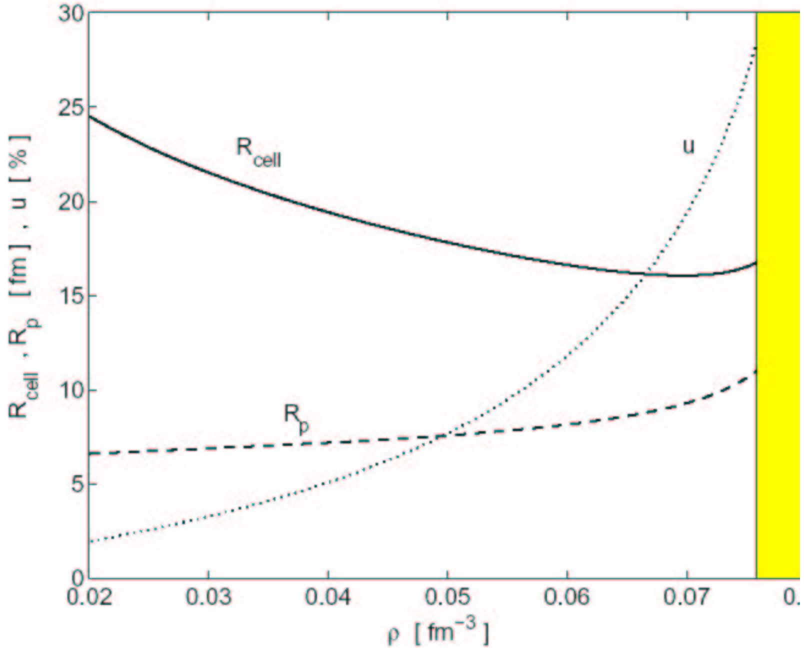
★ Chemical equilibrium

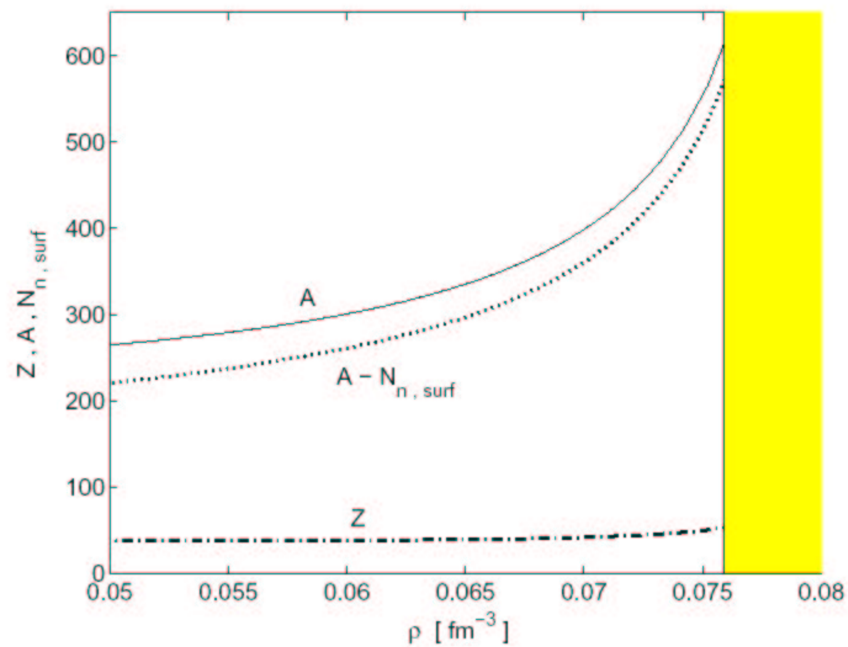
$$\mu_{\mathrm{n},i}^{\mathrm{bulk}} = \mu_{\mathrm{p},o}^{\mathrm{bulk}} = \mu_{\mathrm{n},s}^{\mathrm{bulk}}$$

$$\mu_{\mathrm{n},i}^{\mathrm{bulk}} - \mu_{\mathrm{p},i}^{\mathrm{bulk}} - \mu_e = \frac{8\pi}{5} e^2 n_{\mathrm{p},i} {R_{\mathrm{p}}}^2 \left(1 - \frac{3}{2} u^{1/3} + \frac{1}{2} u\right)$$

Structure of the inner crust within the CLDM

Douchin & Haensel, Phys. Lett. B 484 (2000) 107.





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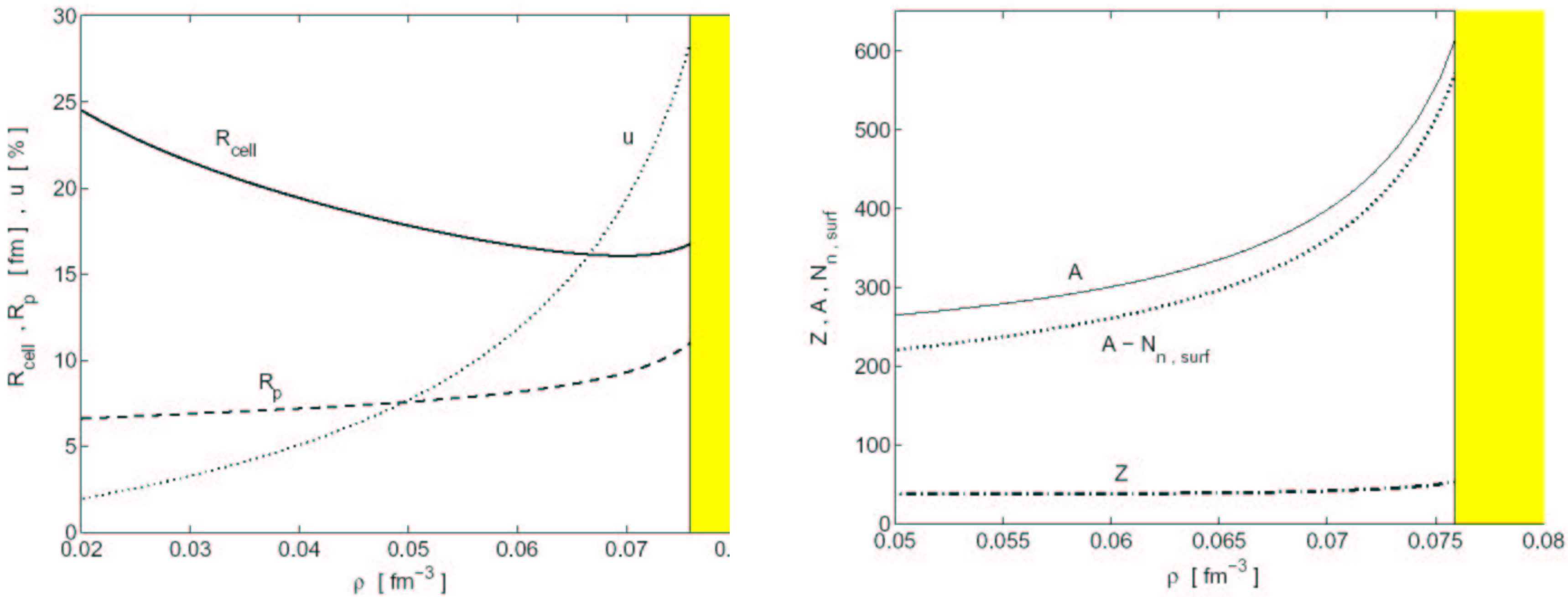

$\Rightarrow$ Z nearly constant throughout the crust

Pasta phases

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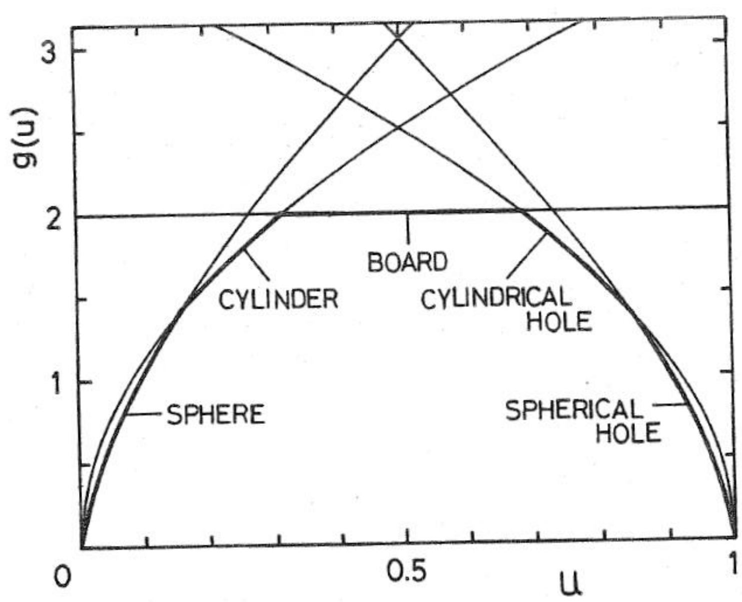

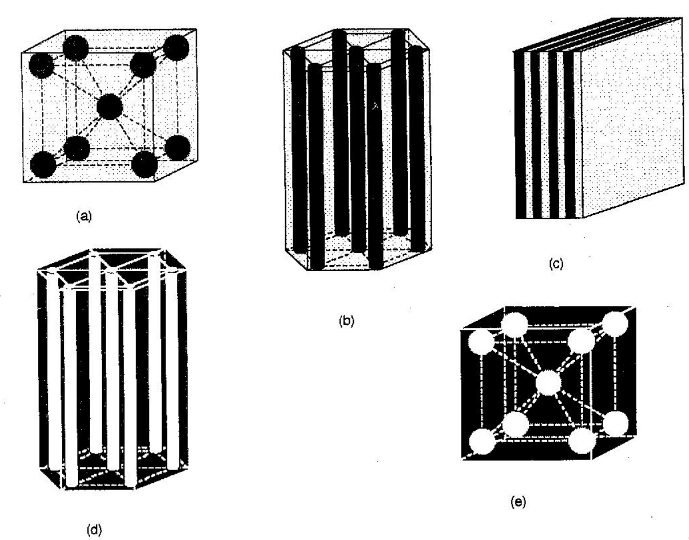

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★ cooling (possiblity of direct URCA processes)
Gusakov *et al.*, Astron. & Astrophys. 421 (2004), 1143.

From liquid drop models to quantum calculations

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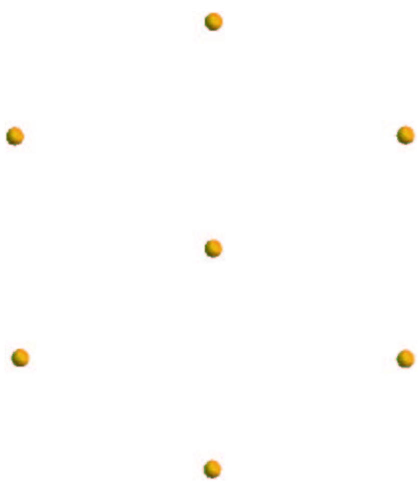

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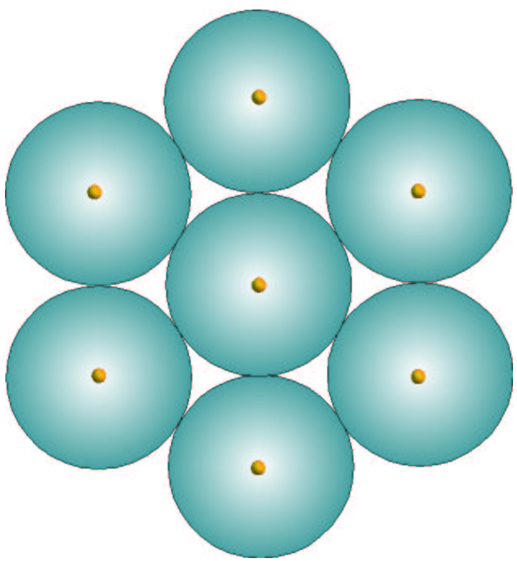

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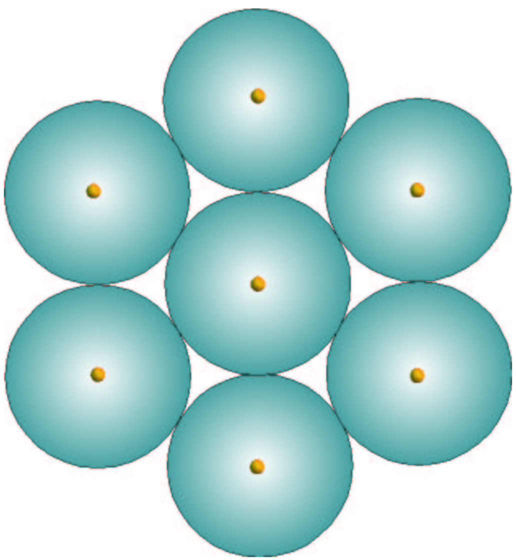

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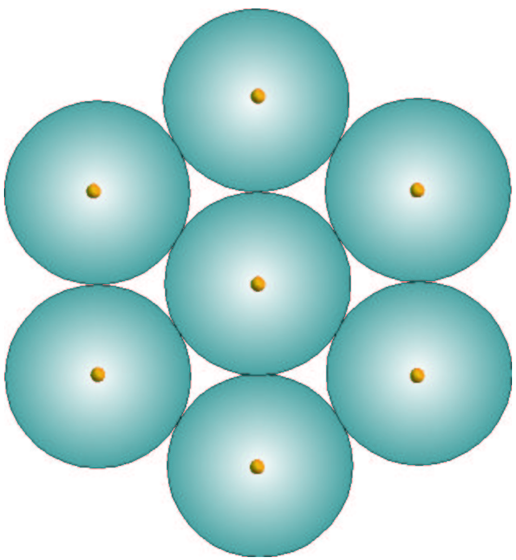
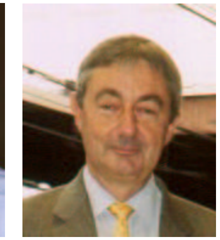


one nucleus in a sphere + arbitrary boundary conditions
$\Rightarrow$ 1D problem

Negele & Vautherin paper

Negele & Vautherin, Nucl. Phys. A207 (1973) 298.



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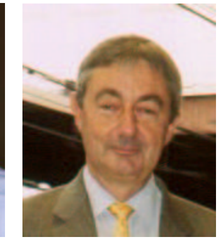

Density matrix expansion (no pairing correlations)

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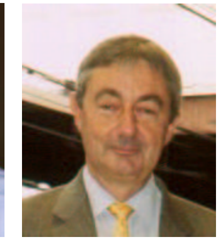

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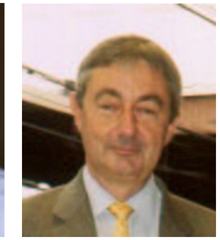

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$$n_q\{\mathbf{r}\} = \sum_{\alpha}^{\mathrm{occ}} |\varphi_\alpha^{(q)}\{\mathbf{r}\}|^2$$

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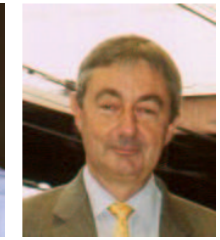

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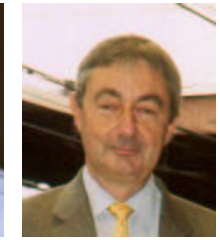

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Boundary conditions :

- ★ wave functions with even l vanish and the radial derivative of those with odd l vanish on the W-S sphere
- ★ averaging of the densities at the cell edge.

Equilibrium structure of the inner crust (T=0)

Spin-orbit coupling terms for neutrons are neglected. Electrons are treated as a uniform relativistic gas.

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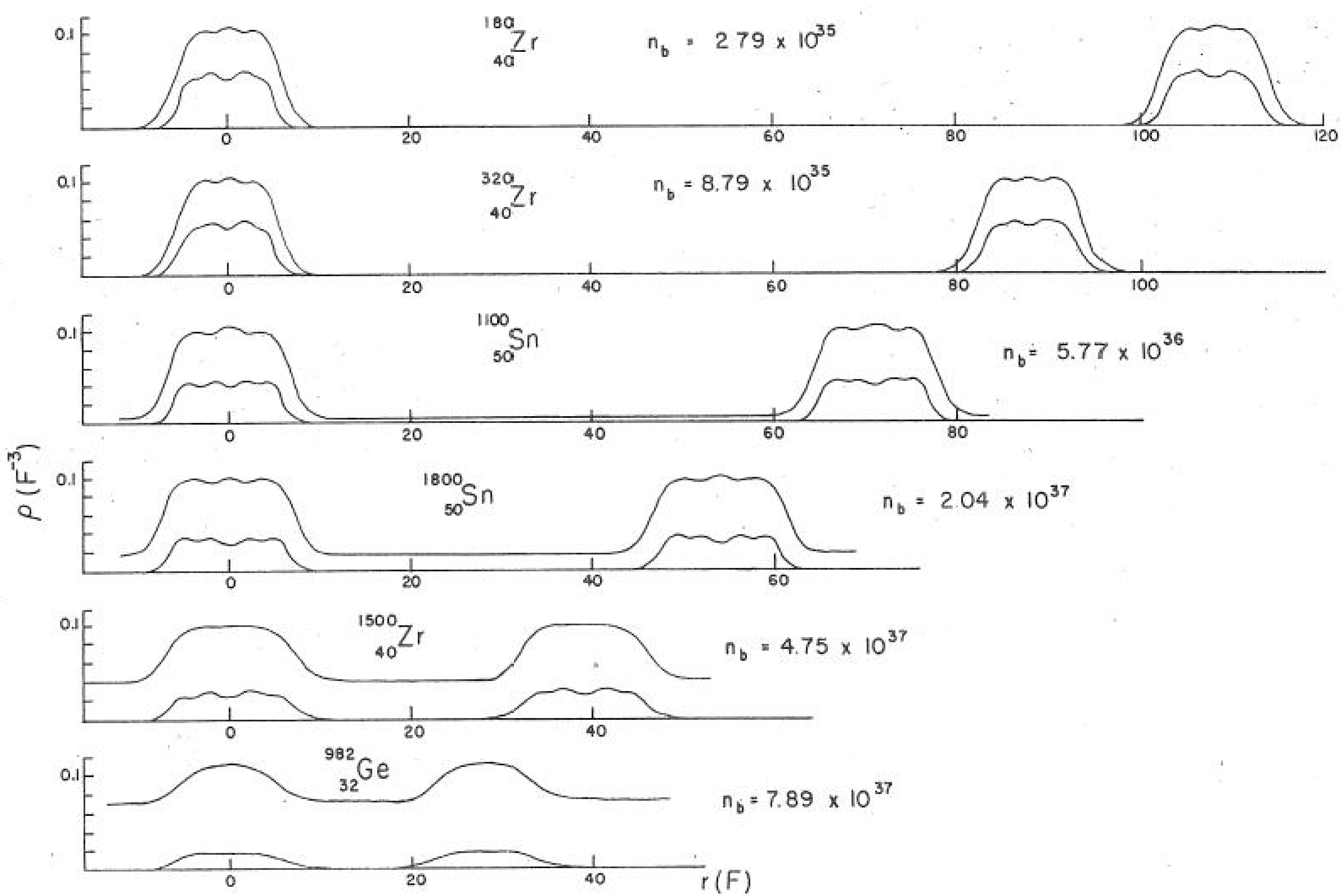

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2.79×10^{35}	140	40	0.2	−26.8	4×10^{-5}	0.53	−1.425	0.436
4.00×10^{35}	160	40	0.3	−29.4	9.7×10^{-5}	0.53	−0.962	0.543
6.00×10^{35}	210	40	0.6	−29.5	2.6×10^{-4}	0.53	−0.462	0.692
8.79×10^{35}	280	40	1.0	−28.5	4.8×10^{-4}	0.53	−0.050	0.865
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3.73×10^{36}	900	50	2.6	−33.6	3.0×10^{-3}	0.46	1.465	1.926
5.77×10^{36}	1050	50	3.3	−34.5	4.7×10^{-3}	0.45	1.996	2.408
8.91×10^{36}	1300	50	4.2	−35.8	7.8×10^{-3}	0.44	2.610	2.981
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4.75×10^{37}	1460	40	10.9	−54.0	4.36×10^{-2}	0.28	6.428	6.660
7.89×10^{37}	950	32	15.0	−68.3	7.37×10^{-2}	0.16	8.611	8.657

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$\Rightarrow$ strong proton shell effects at Z=40 (Zr) and Z=50 (Sn)

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n_b (cm^{-3})	N	Z	μ_n (MeV)	μ_p (MeV)	$\tilde{\rho}_G$ (fm^{-3})	$\tilde{x}$	$(E/A)-m_n$ (MeV)	$(E_{gas}/A)-m_n$ (MeV)
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4.00×10^{35}	160	40	0.3	−29.4	9.7×10^{-5}	0.53	−0.962	0.543
6.00×10^{35}	210	40	0.6	−29.5	2.6×10^{-4}	0.53	−0.462	0.692
8.79×10^{35}	280	40	1.0	−28.5	4.8×10^{-4}	0.53	−0.050	0.865
1.59×10^{36}	460	40	1.4	−29.4	1.2×10^{-3}	0.52	0.541	1.214
3.73×10^{36}	900	50	2.6	−33.6	3.0×10^{-3}	0.46	1.465	1.926
5.77×10^{36}	1050	50	3.3	−34.5	4.7×10^{-3}	0.45	1.996	2.408
8.91×10^{36}	1300	50	4.2	−35.8	7.8×10^{-3}	0.44	2.610	2.981
2.04×10^{37}	1750	50	6.5	−43.6	1.84×10^{-2}	0.35	4.097	4.422
4.75×10^{37}	1460	40	10.9	−54.0	4.36×10^{-2}	0.28	6.428	6.660
7.89×10^{37}	950	32	15.0	−68.3	7.37×10^{-2}	0.16	8.611	8.657

$\Rightarrow$ strong proton shell effects at Z=40 (Zr) and Z=50 (Sn)
$\Rightarrow$ unlike lattice spacing, the nuclear size is almost constant throughout the crust

Superfluidity

Superfluidity

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  - ⋆ Superfluidity predicted by Migdal in 1959 before the discovery of pulsars

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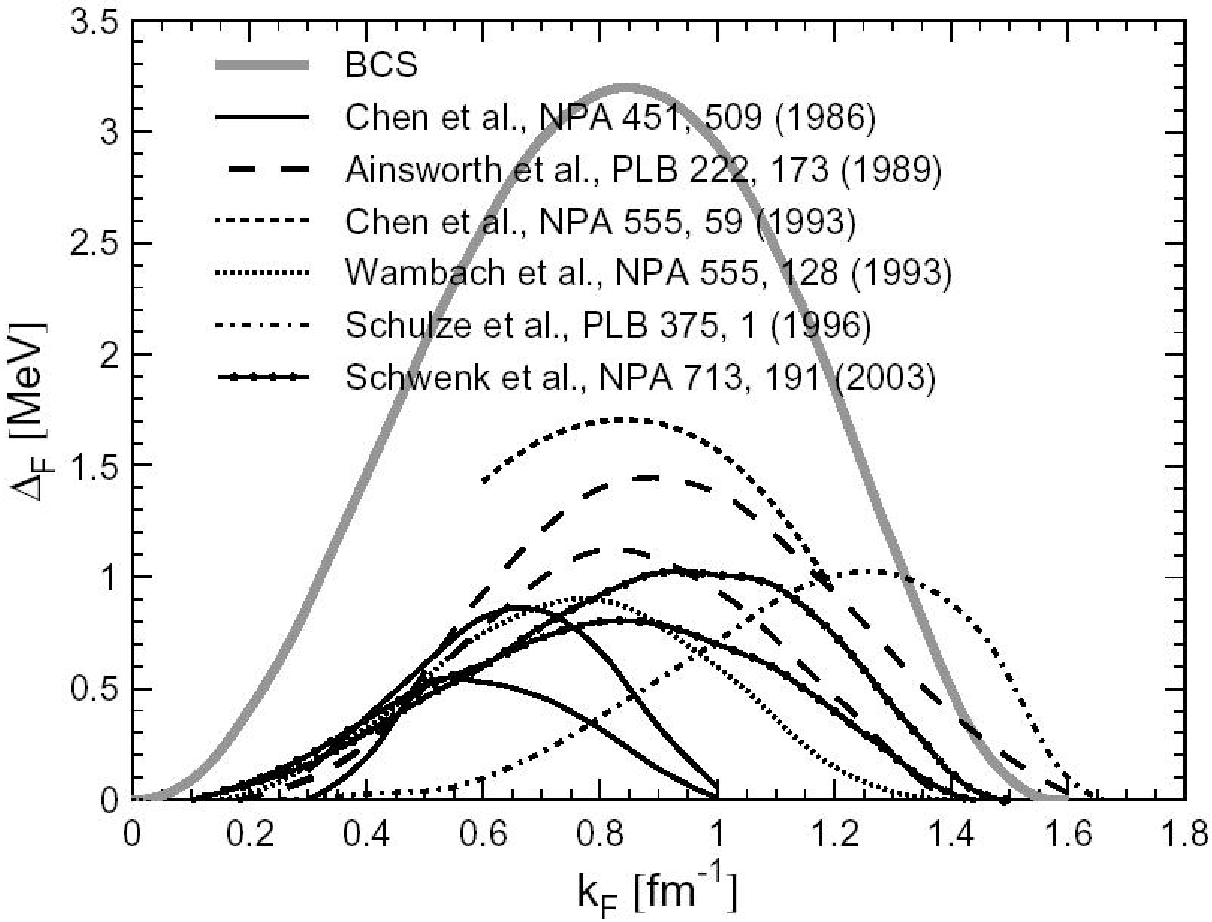
- ⋆ Evidence of superfluidity from long relaxation times after pulsar glitches
- ⋆ Superfluidity involved in some mechanism of glitches (vortex pinning)
- ⋆ Superfluidity in the crust affects the cooling time

Superfluidity in low density pure neutron matter

Baldo *et al.*, NPA 749 (2005) 42c.



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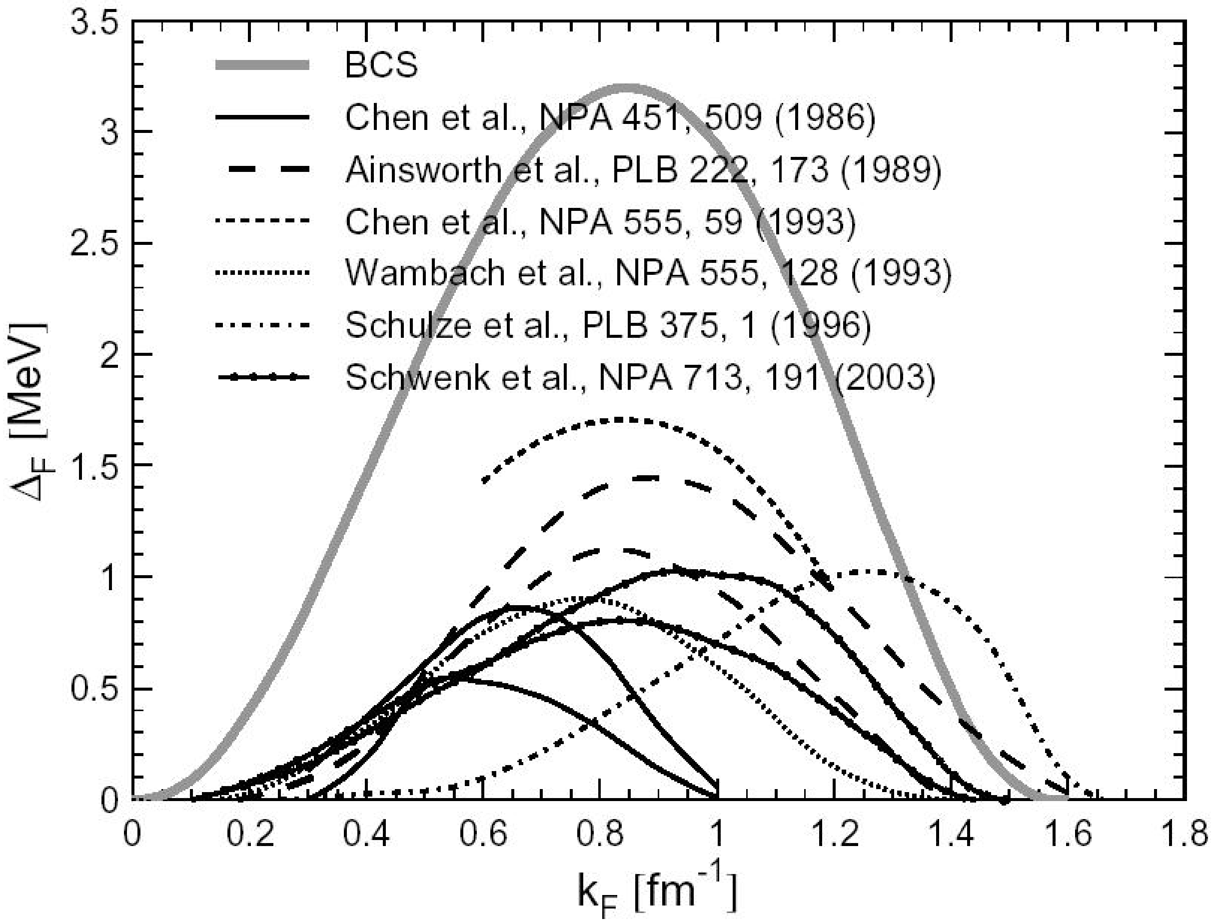

$\Rightarrow$ Pairing gap $\Delta \sim$ MeV

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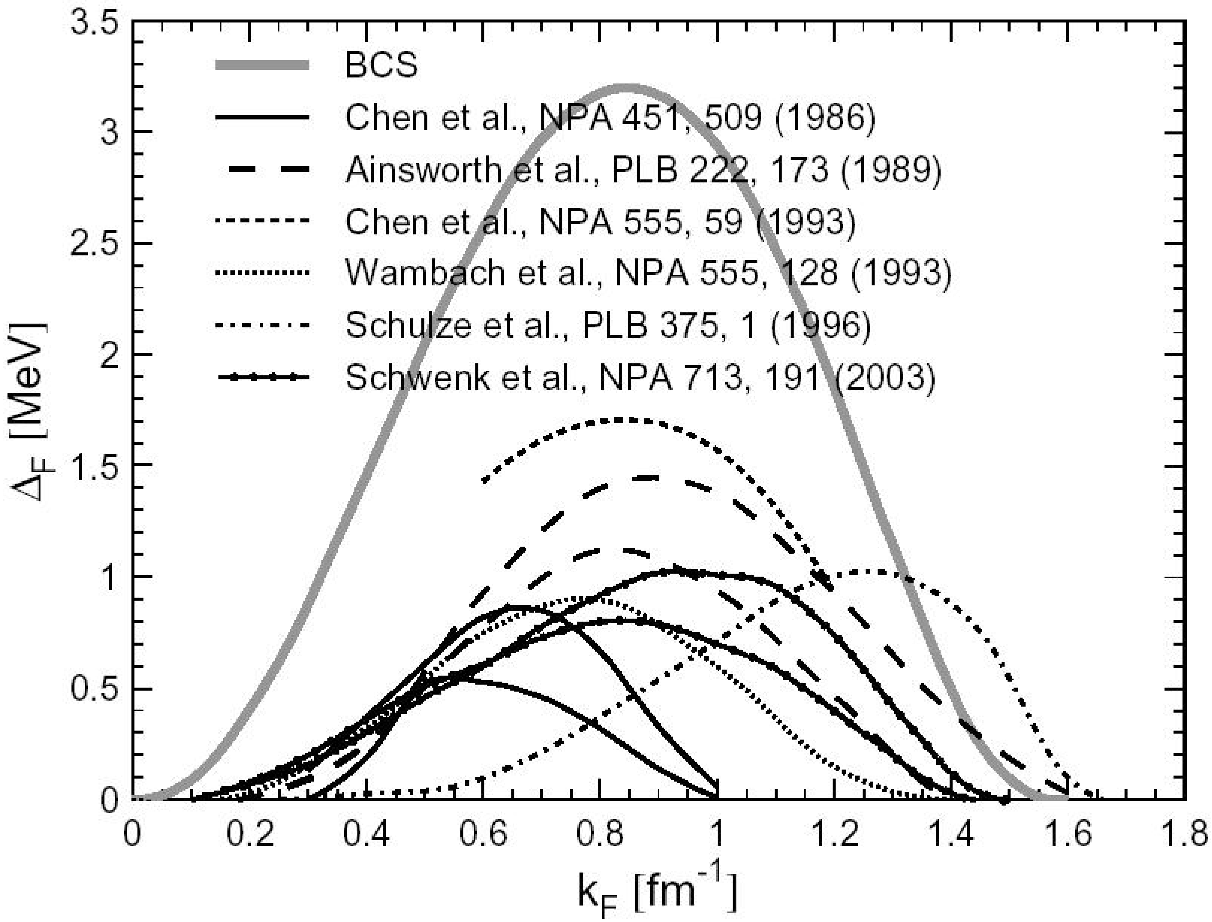

$\Rightarrow$ Pairing gap $\Delta \sim$ MeV

$\Rightarrow$ critical temperature $T_{\rm cn} \sim \Delta/k_{\rm B}$ and density range of superfluidity depend on medium effects

Superfluidity in neutron star crust matter

Sandulescu *et al.*, Phys. Rev. C 69 (2004), 045802.

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★ Mean field approximation : independent pairs of correlated nucleons

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$\Rightarrow$ Hartree-Fock Bogoliubov equations

$$\begin{pmatrix} \mathcal{H}\{\mathbf{r}\} - \mu & \Delta\{\mathbf{r}\} \\ \Delta^*\{\mathbf{r}\} & -\mathcal{H}^*\{\mathbf{r}\} + \mu \end{pmatrix} \begin{pmatrix} u\{\mathbf{r}\} \\ v\{\mathbf{r}\} \end{pmatrix} = E \begin{pmatrix} u\{\mathbf{r}\} \\ v\{\mathbf{r}\} \end{pmatrix}$$

with Skyrme (SLy4) nucleon-nucleon interactions and with two sets of pairing force (weak/strong)

$$V\{\mathbf{r} - \mathbf{r}'\} = V_0 \left(1 - \eta \left(\frac{n\{\mathbf{r}\}}{n_0}\right)^\gamma \right) \delta\{\mathbf{r} - \mathbf{r}'\}$$

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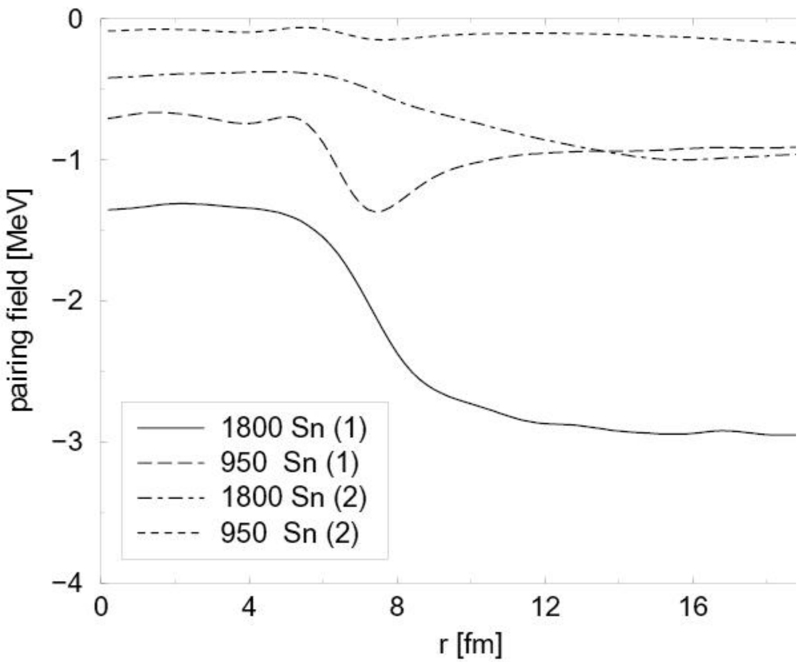
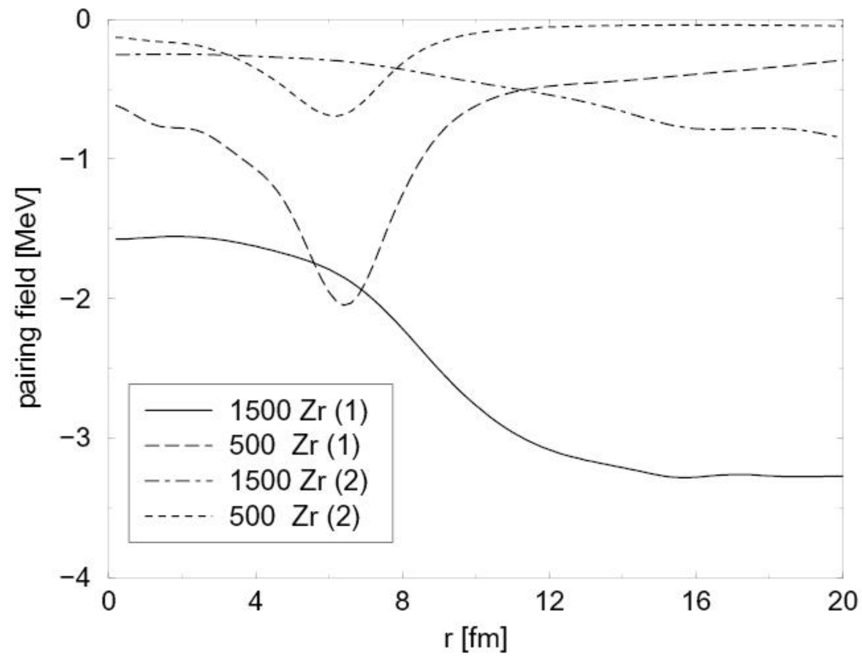
★ $R_{\rm cell}$, N and Z taken from N&V

Pairing field





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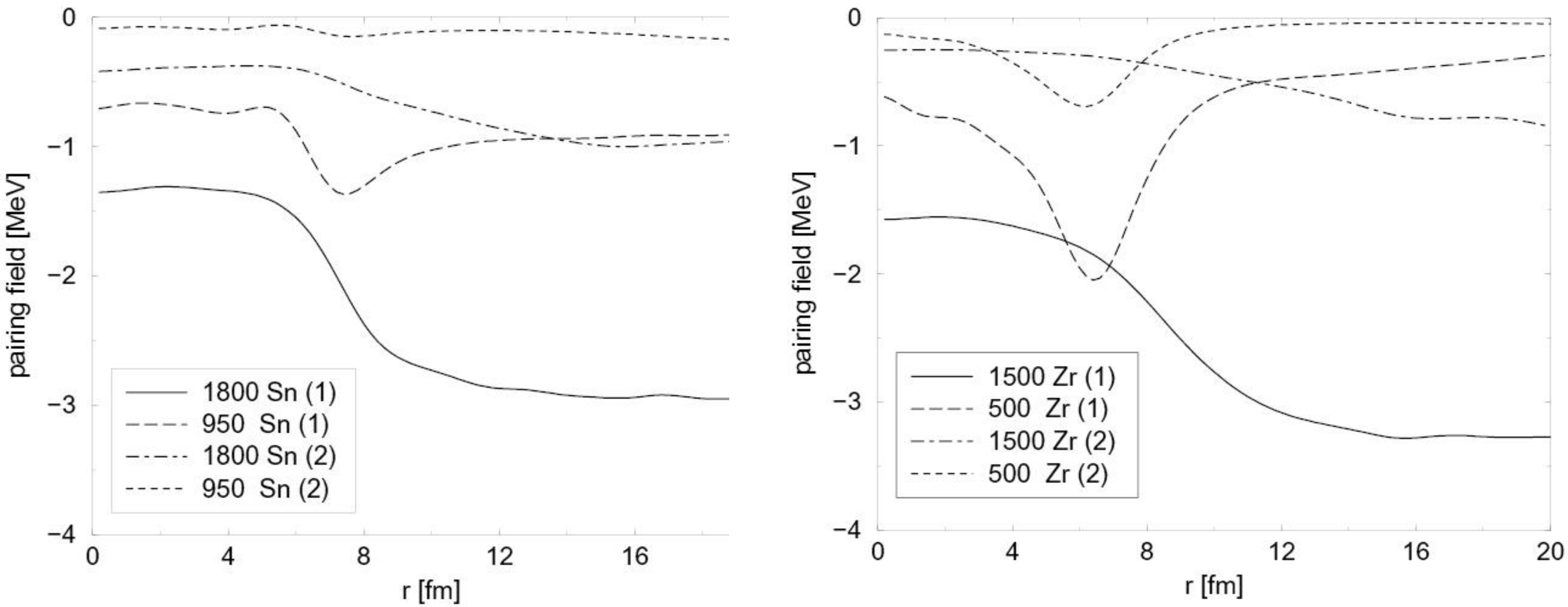

⇒ The pairing field varies smoothly between the nuclei and the neutron gas : proximity effect
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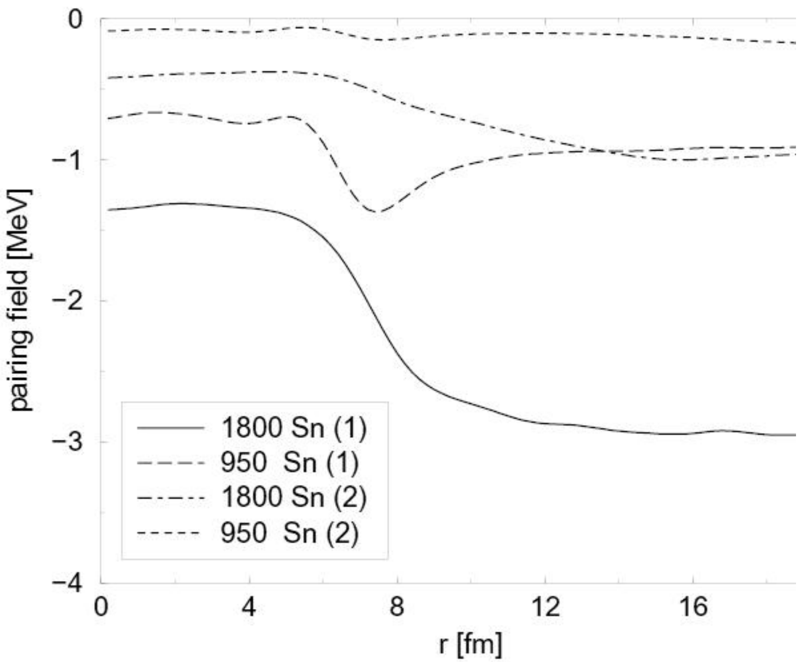

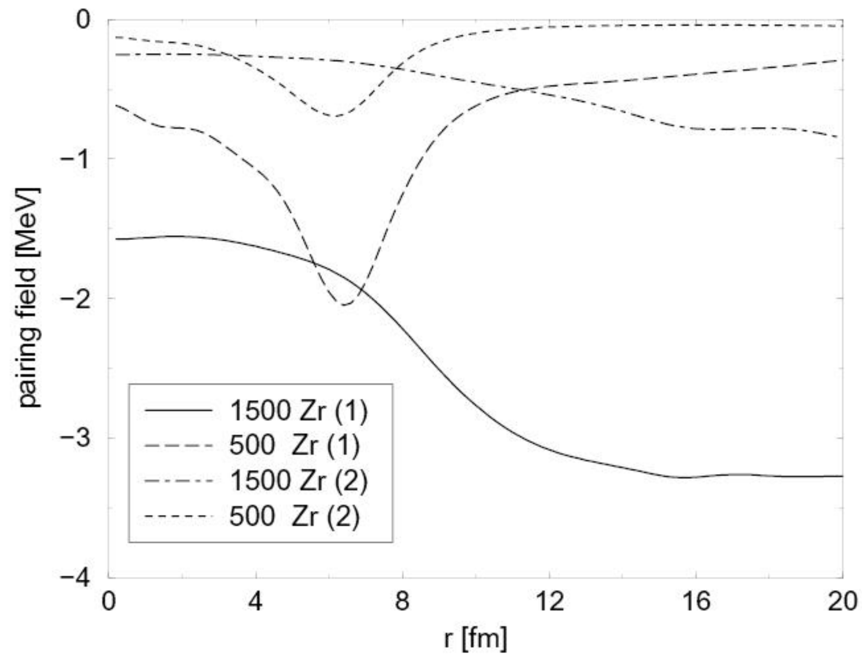

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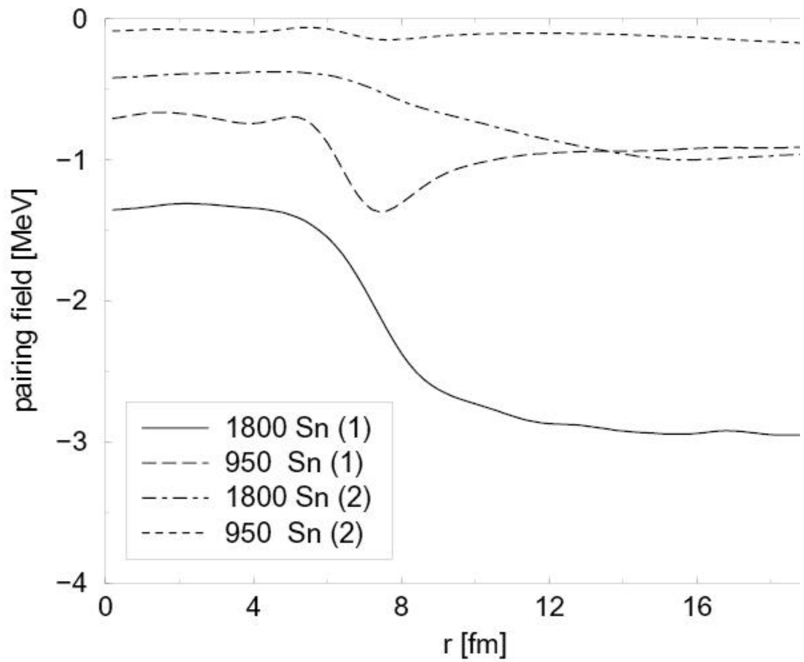

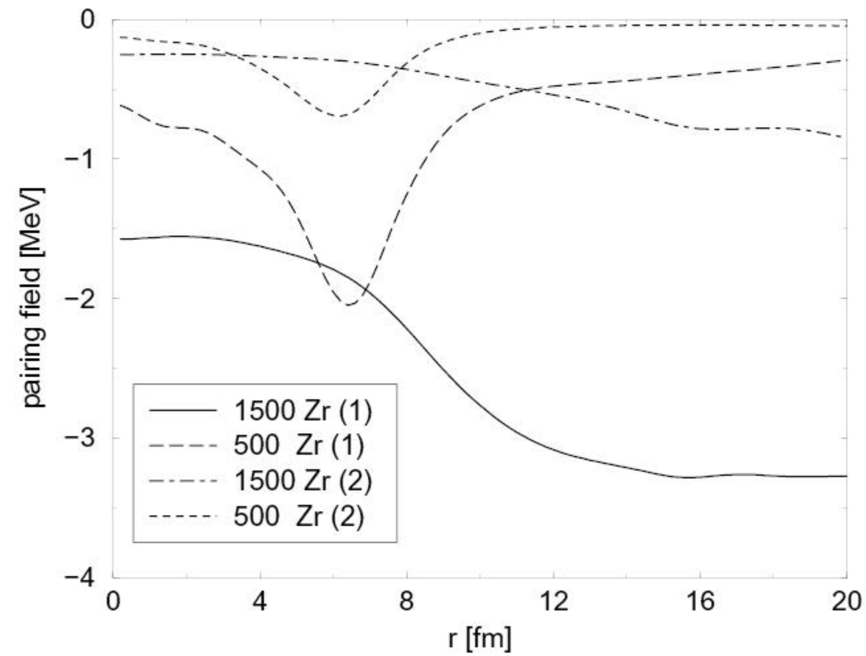

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⇒ The pairing field is very sensitive to the pairing force

Superfluidity and cooling

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HFB calculations with a fixed Woods-Saxon mean field parametrised from the results of N & V.

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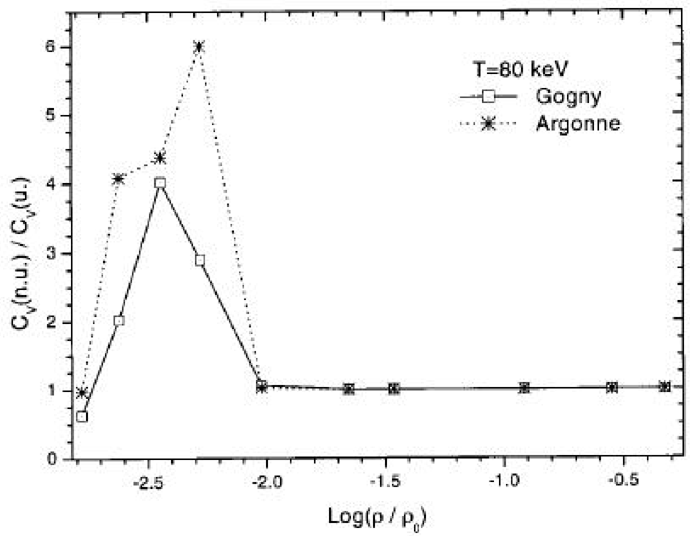

FIG. 12*a*

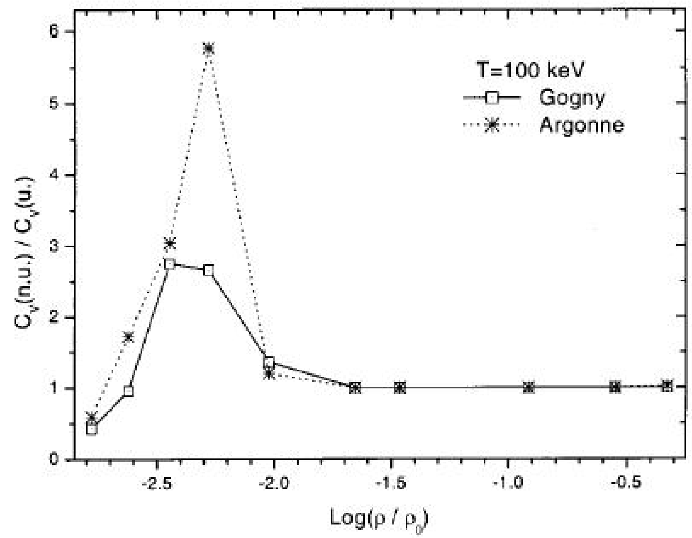

FIG. 12*b*

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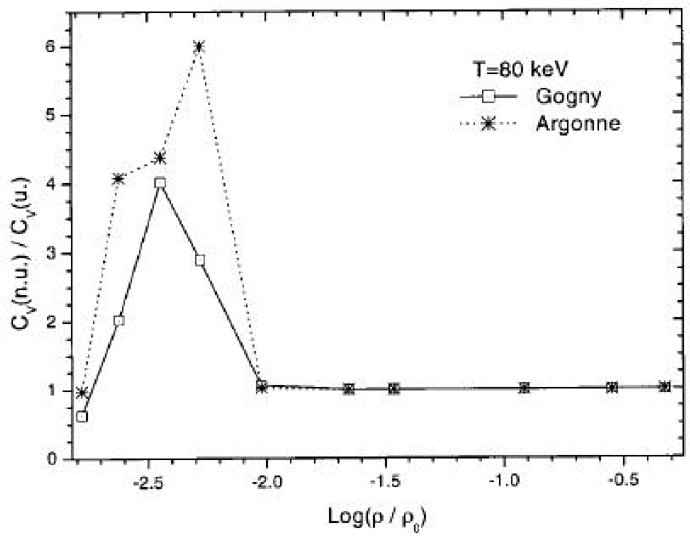

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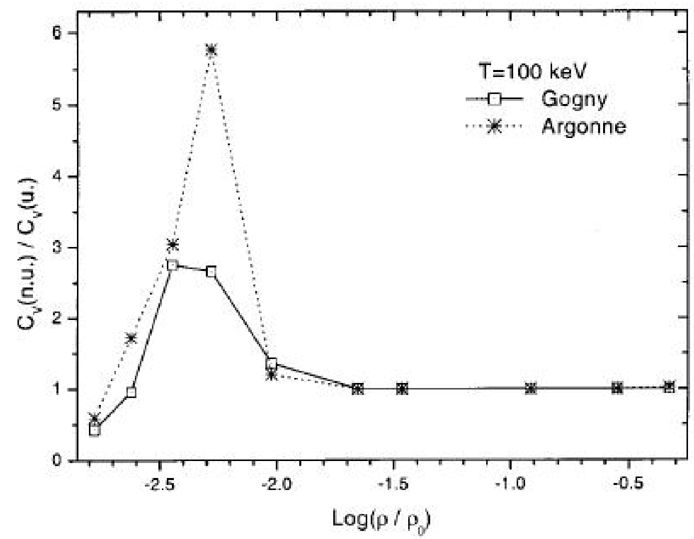

FIG. 12*b*

$\Rightarrow$ the specific heat is significantly increased at low density due to the suppression of the pairing field inside nuclei

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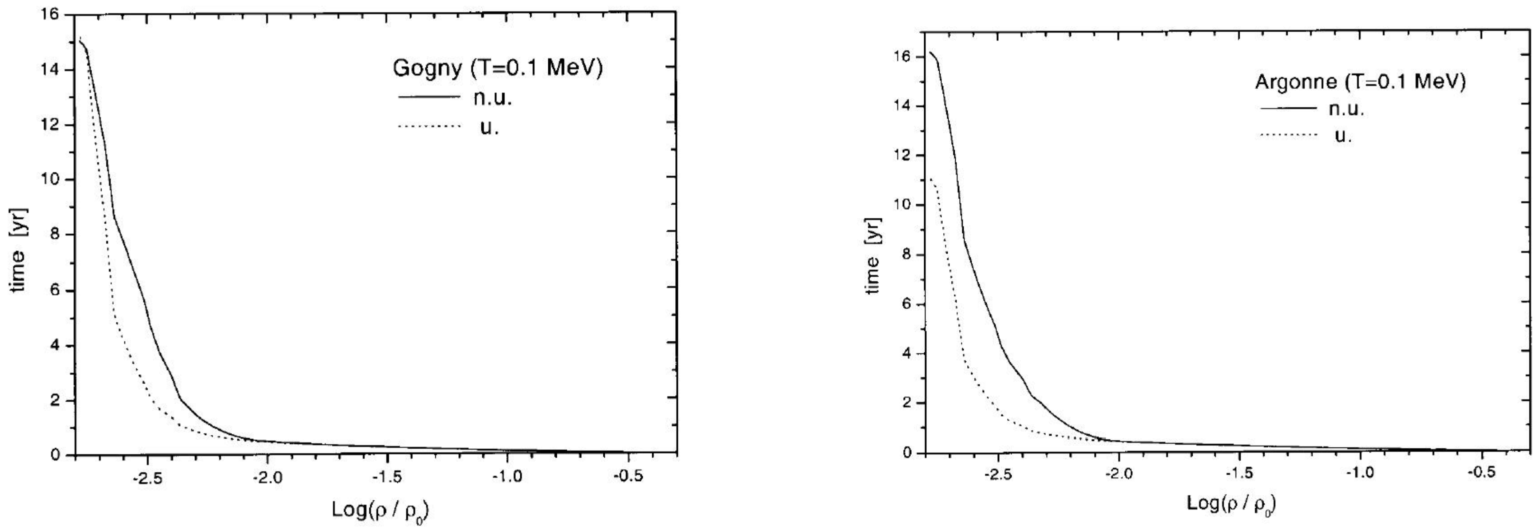

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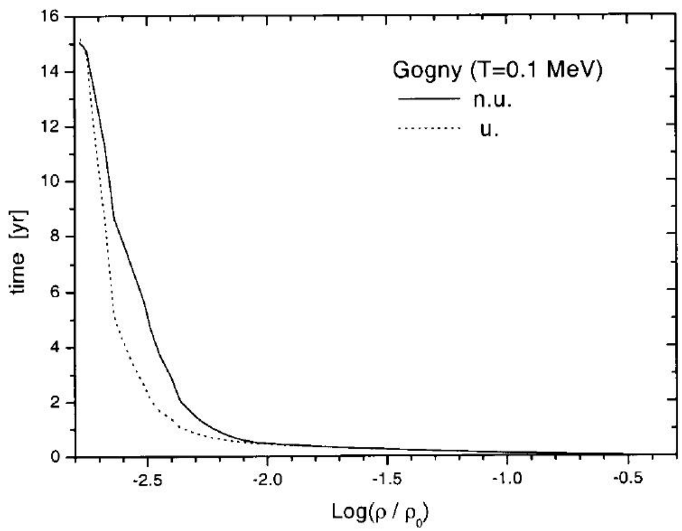
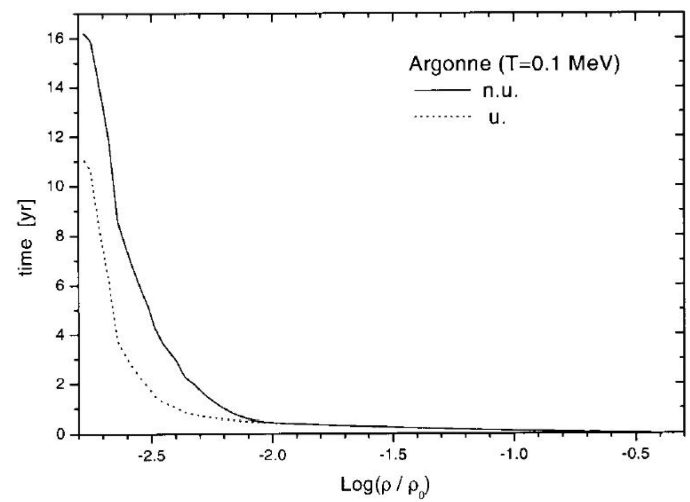
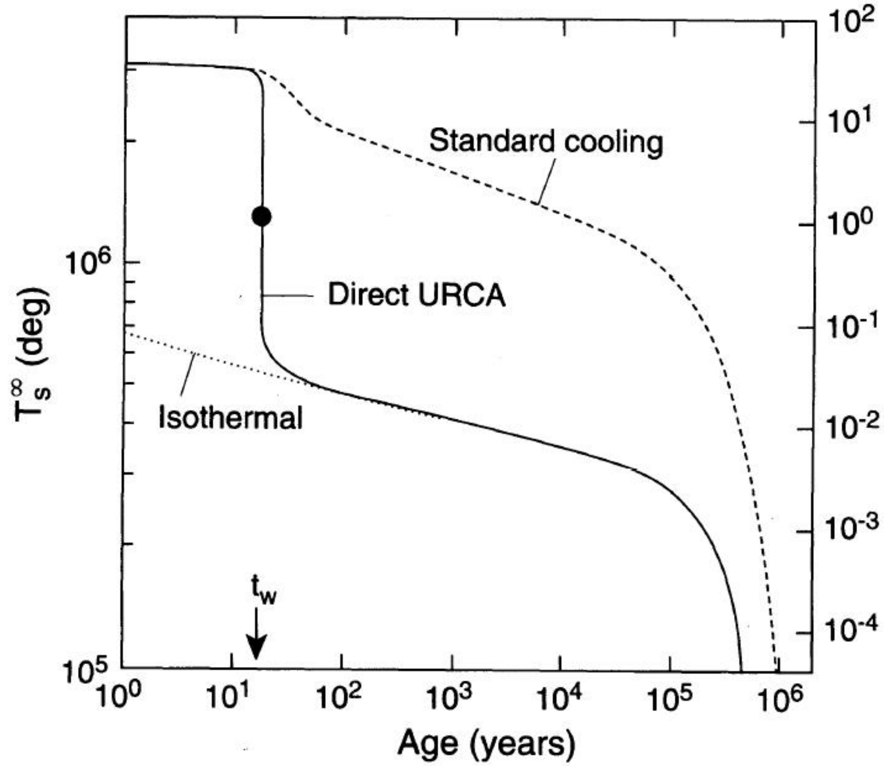




⇒ the presence of the nuclear lattice tend to increase the heat diffusion time along the inner crust therefore the surface temperature

Observational constraints on the crust

Lattimer *et al.*, Astrophys.. J. 425 (1994), 802.



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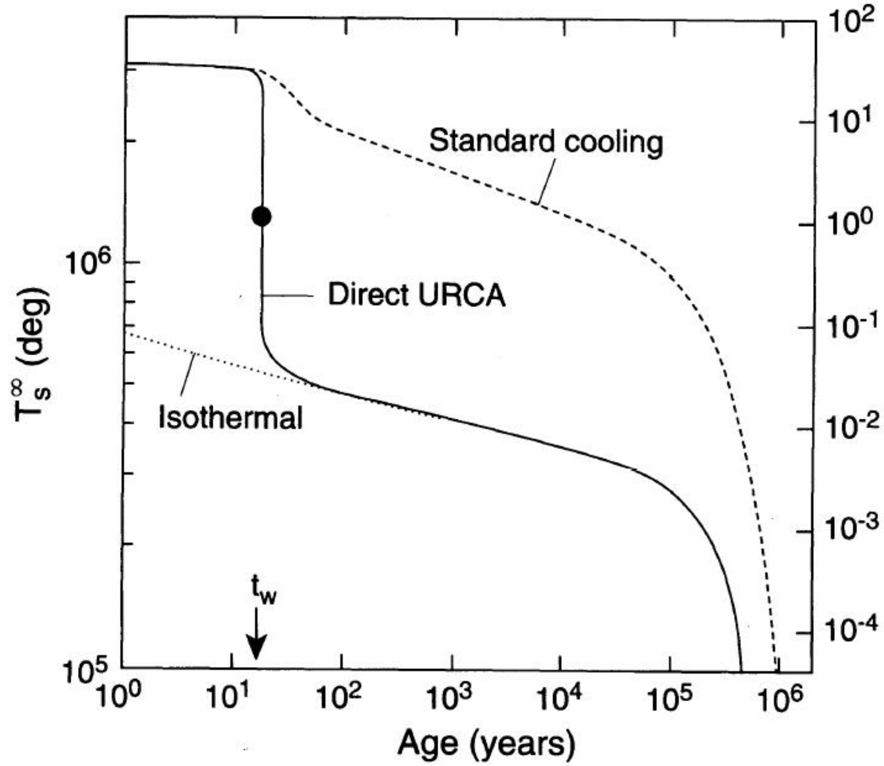

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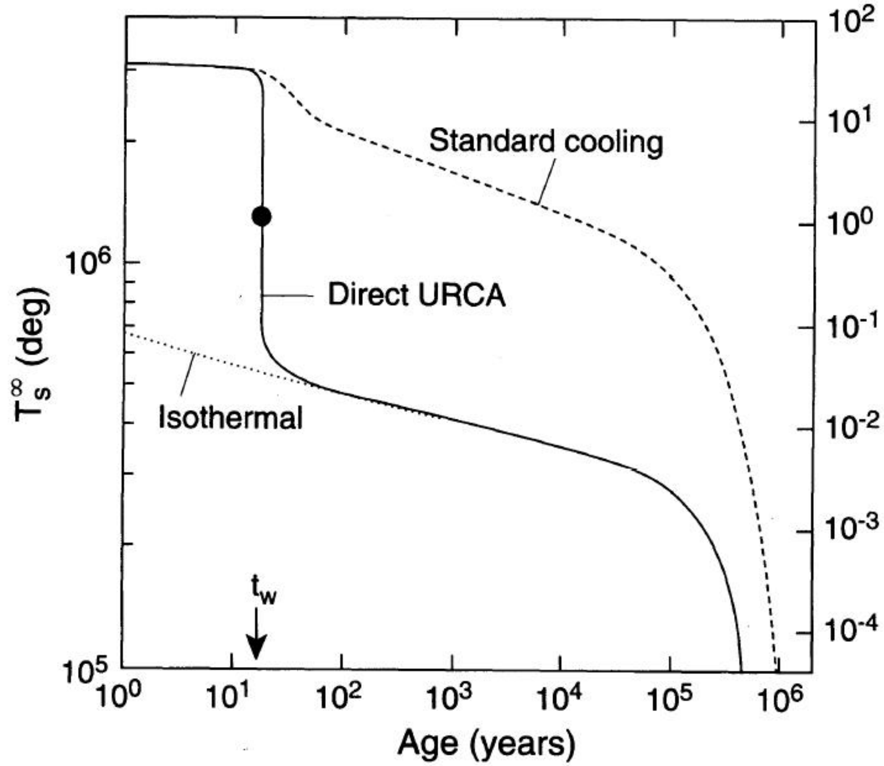

$\Rightarrow t_w$ is related to the diffusion of heat in the crust
$\Rightarrow$ Measures of t_w from observations can constrain models of neutron star crust

Effects of pairing on the structure of the crust ?

Baldo *et al.*, Nucl. Phys. A 750 (2005) 409.
Ground state of the crust in the regions of maximal neutron pairing ($\rho \simeq 1.9 \times 10^{13}$ g.cm^{-3}) ?

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$$\mathcal{F}\{r\} = \left(1+\exp\{(r-R_m)/d_m\}\right)^{-1}, \quad n_{\rm p}\{R_m\} = 0.1 n_{\rm p}\{0\}$$

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+W-S approximation

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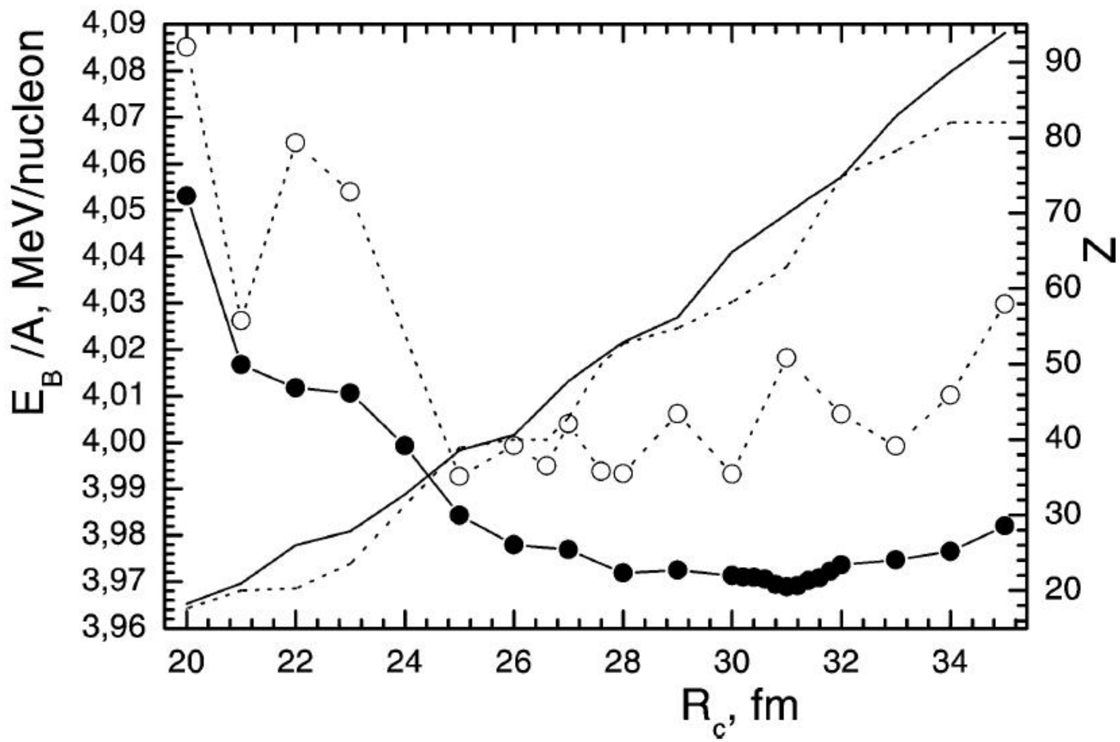

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Equilibrium structure of the crust ?
Fayan's functional only

- ★ with pairing $Z = 71$ and $R_{\mathrm{cell}} = 31$ fm
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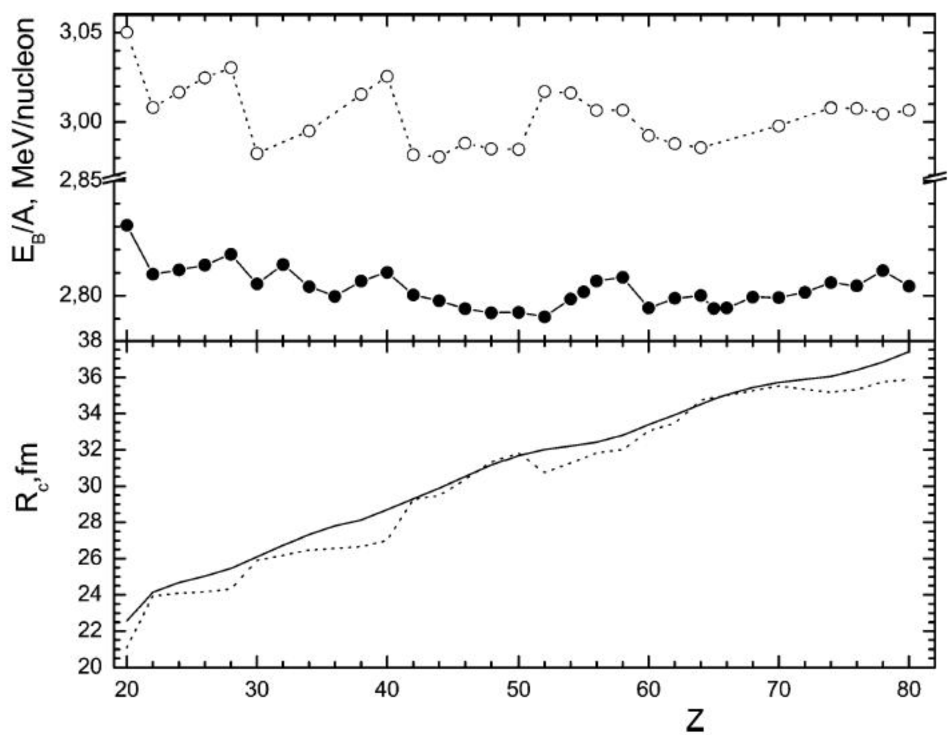

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$\Rightarrow$ the composition of the nuclei is strongly affected

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Equilibrium structure of the crust ?
Fayan's functional only

- ★ with pairing $Z = 71$ and $R_{\rm cell} = 31$ fm
- ★ without pairing $Z = 40$ and $R_{\rm cell} = 25$ fm

Generalised functional

- ★ with pairing $Z = 52$ and $R_{\rm cell} = 32$ fm
- ★ without pairing $Z = 44$ and $R_{\rm cell} = 29.5$ fm

$\Rightarrow$ the shell effects are washed out by pairing : no magic number
$\Rightarrow$ the binding energy per nucleon vs Z has a very flat minimum : probably several nuclear species A, Z coexist at a given density
$\Rightarrow$ the composition of the nuclei is strongly affected
$\Rightarrow$ but results are very sensitive to the energy functional !

Necessity to go beyond the W-S approximation

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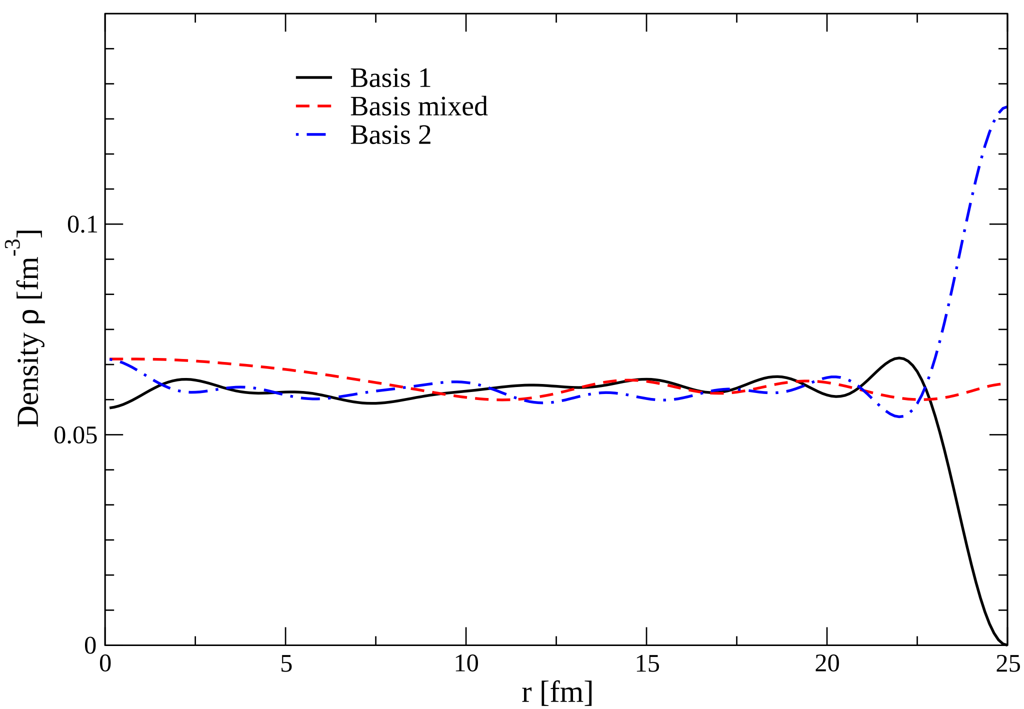

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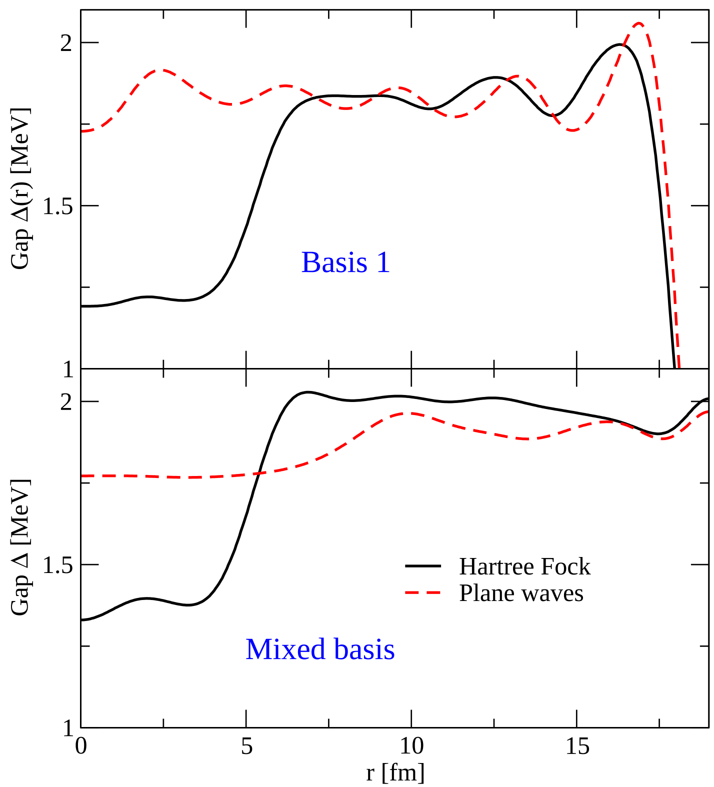

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necessity for reconsidering boundary conditions more rigorously !

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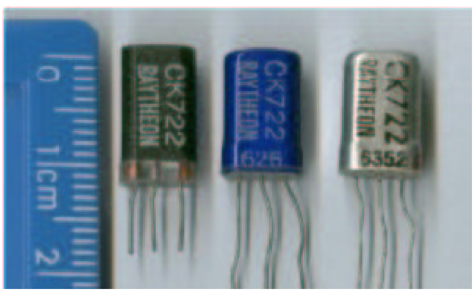

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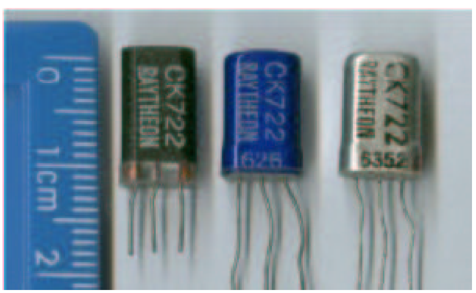

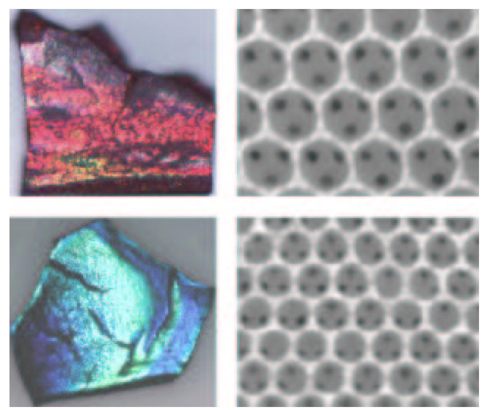

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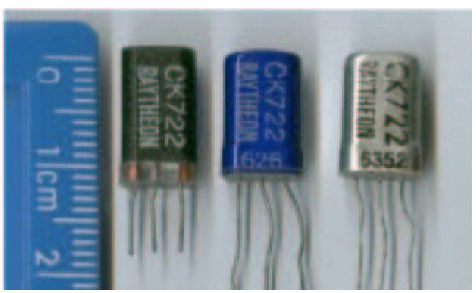

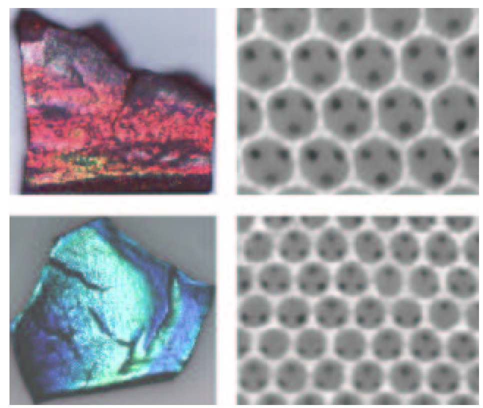

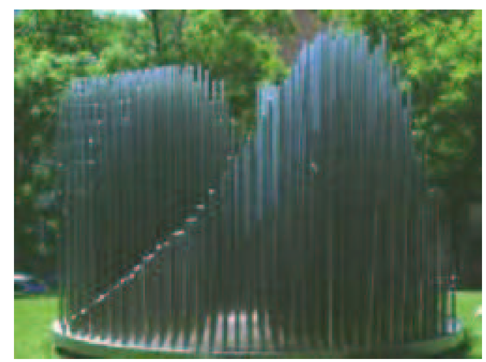

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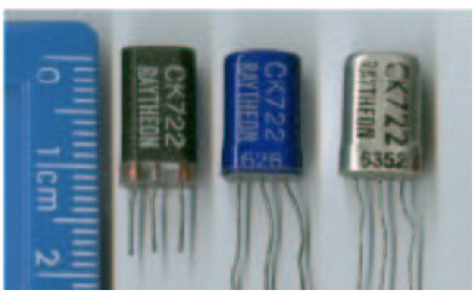

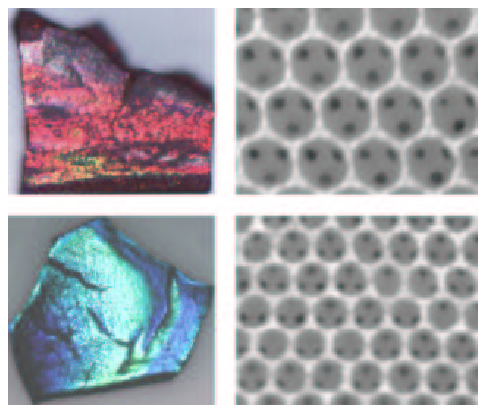

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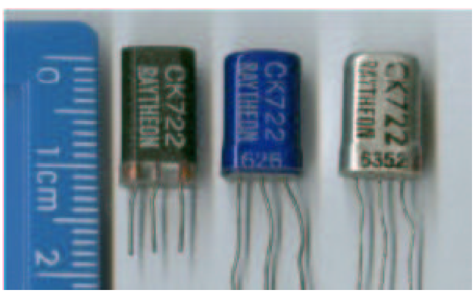

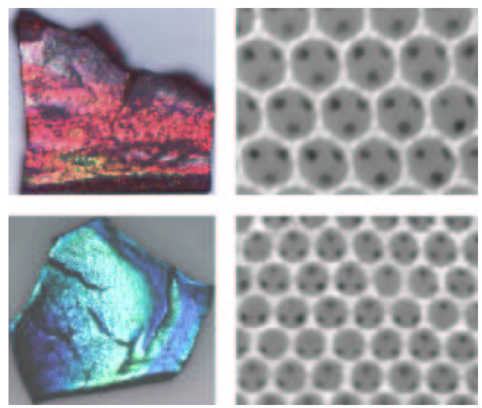


$\Rightarrow$ “neutronic” crystals

Go beyond the W-S approximation by including Bragg scattering of dripped neutrons by crustal nuclei

From solid state to nuclear physics

Analogy between dripped neutrons in NS crust and conduction electrons in solids

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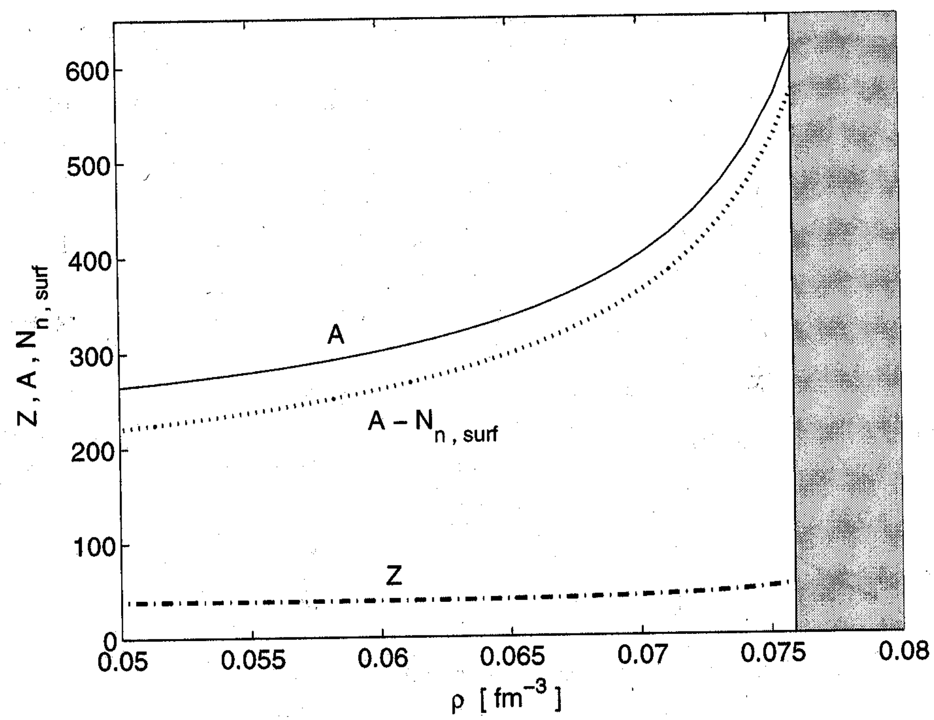

Douchin & Haensel, Phys.Lett. B485 (2000) 107

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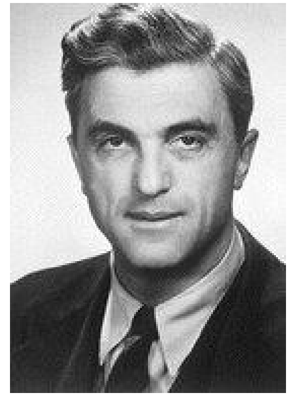
Floquet (1883)-Bloch(1928) theorem



Single particle states in a periodic medium ?

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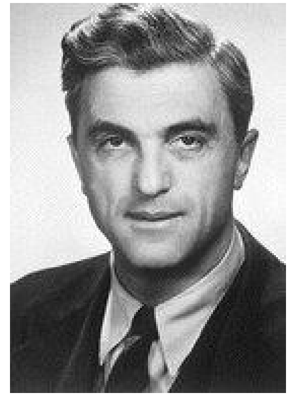

Single particle states in a periodic medium ?
$\Rightarrow$ modulated plane wave (« fonctions périodiques de seconde espèce »)

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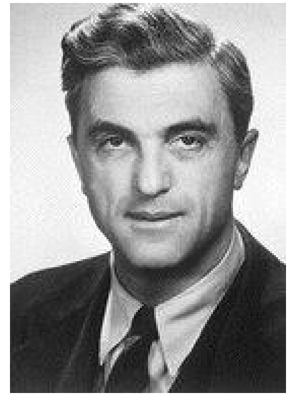

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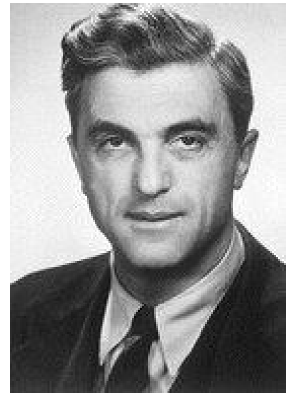

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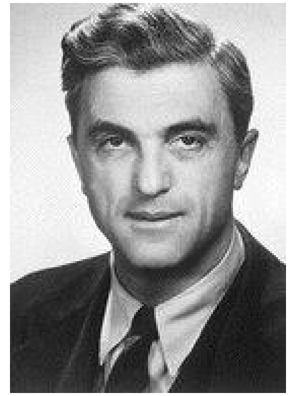

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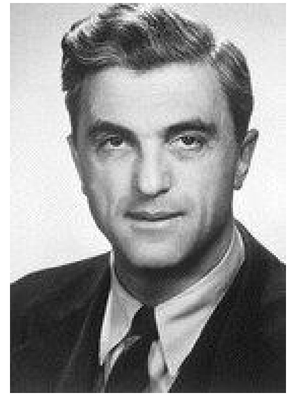

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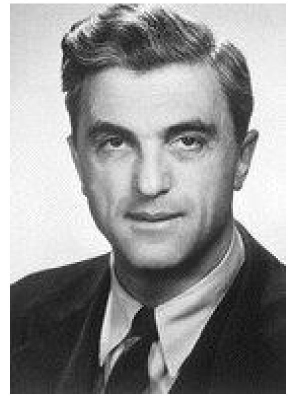

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$\Rightarrow$ local and global symmetries are both included !

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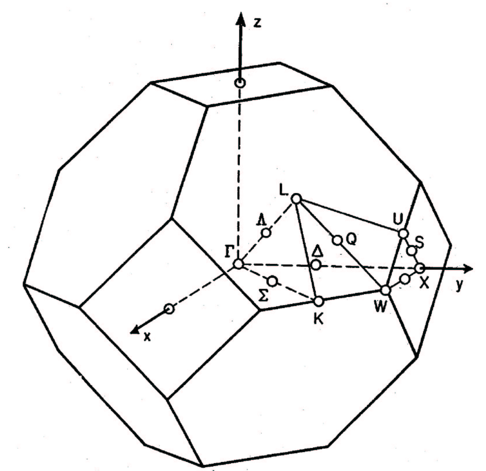

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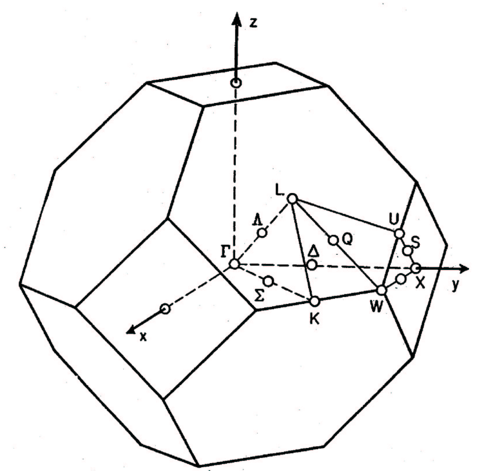

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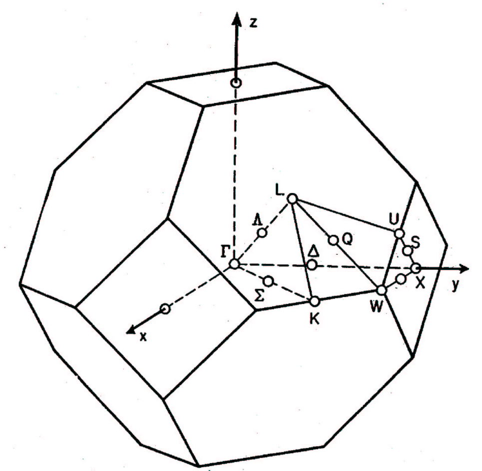

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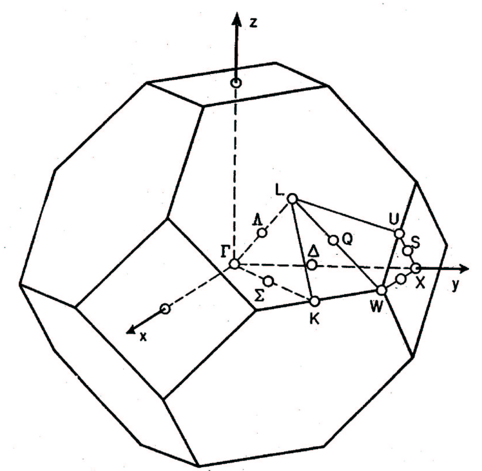

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Prescription of Magierski *et al* $\Rightarrow$ only $k=0$ solutions

Back to the W-S approximation

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E. P. Wigner & F. Seitz, Phys. Rev. 43 (1933), 804.

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« *It will be quite a good approximation to replace the polyhedron by a sphere of equal volume, and to take as boundary conditions that the derivative of the wave function vanishes at the boundary of this sphere.* »

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Energy bands possess a lattice symetry

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The set of all $\mathbf{K} \Rightarrow$ reciprocal lattice

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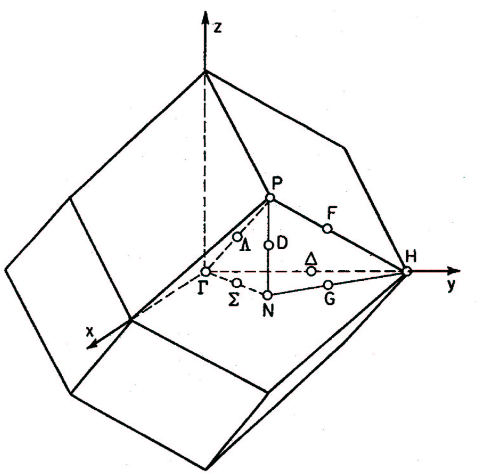

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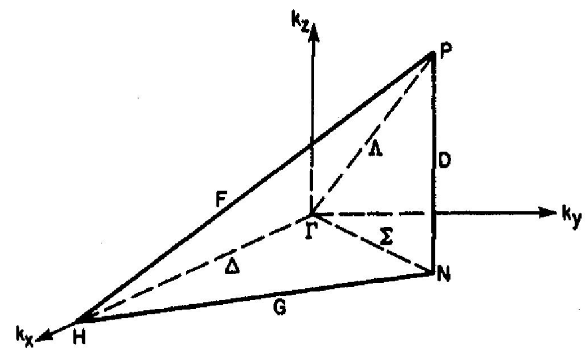
Example : bcc lattice



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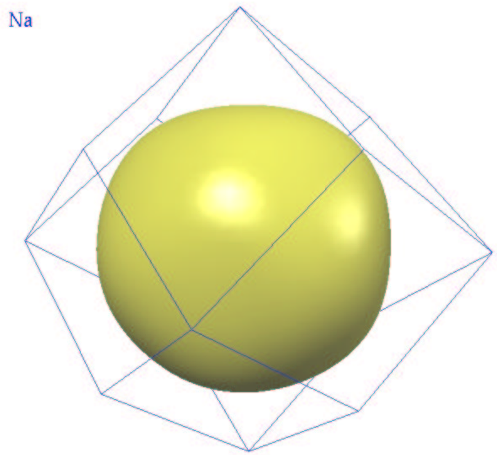
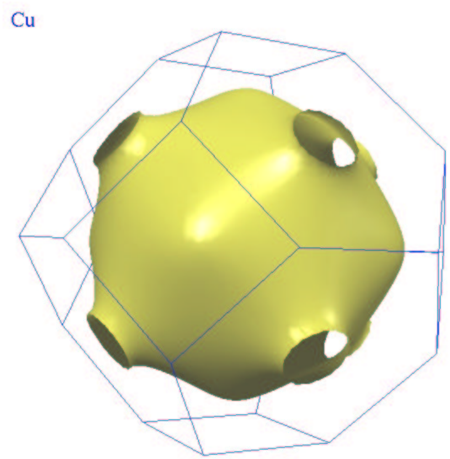
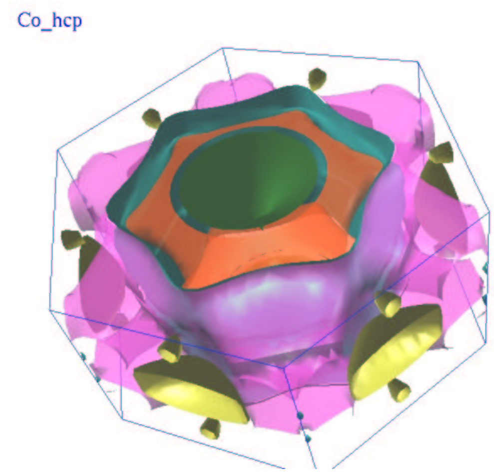
In the empty lattice (uniform) limit , the Fermi surface is a sphere

Example of Fermi surfaces

Examples in solid state physics : Sodium (bcc), Copper (fcc) and Cobalt (hcp)







http ://www.phys.ufl.edu/fermisurface/

Landau-Luttinger theorem : $\mathcal{V}_{\mathrm{F}} = (2\pi)^3 n_{\mathrm{n}}$

J. M. Luttinger, Phys. Rev. 119 (1960), 1153.

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Bloch theorem ensures that the density possesses the full crystal symmetry

Neutron density and symmetries

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Neutron star crust in the neutron drip region

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Hartree-Fock calculation with Skyrme (SLy4) effective nucleon-nucleon interactions

$$-\nabla\cdot\frac{\hbar^2}{2m_{\mathrm{n}}^{\oplus}\{\mathbf{r}\}}\nabla\varphi_{\mathbf{k}}\{\mathbf{r}\}+U_{\mathrm{n}}\{\mathbf{r}\}\varphi_{\mathbf{k}}\{\mathbf{r}\}-\mathrm{i}\mathbf{W}_{\mathrm{n}}\{\mathbf{r}\}\cdot\nabla\times\sigma\varphi_{\mathbf{k}}\{\mathbf{r}\} = \mathcal{E}\varphi_{\mathbf{k}}\{\mathbf{r}\}$$

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Equilibrium lattice spacing and nuclear composition taken from Negele & Vautherin (1973)

- ★ Body centered cubic lattice
- ★ W-S sphere radius $R_{\rm cell}\simeq$ 54.1 fm
- ★ $n_{\rm n}\{\mathbf{r}\}$ and $n_{\rm p}\{\mathbf{r}\}$ from N&V + ETF $\Rightarrow m_{\rm n}{}^{\oplus}\{\mathbf{r}\}$, $U_{\rm n}\{\mathbf{r}\}$ and $\mathbf{W}_{\rm n}\{\mathbf{r}\}$

Linearised Augmented Plane Wave method

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Andersen (1975) from the idea of Slater (1937).

Linearised Augmented Plane Wave method

Variational method : $\varphi_{\mathbf{k}}\{\mathbf{r}\} = \sum_{\alpha} c_{\alpha}\phi_{\alpha}\{\mathbf{r}\}$

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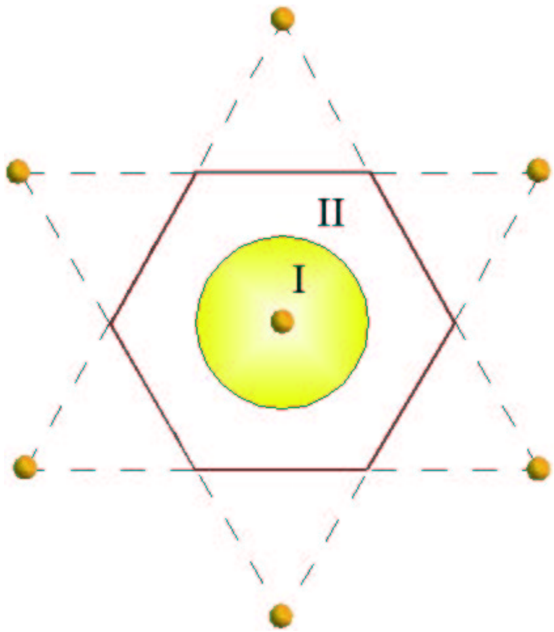

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$u_l\{\mathcal{E}_l, r\}$ radial solution for fixed $\mathcal{E}_l$

$$-\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r} r^2 \frac{\hbar^2}{2m_{\mathrm{n}^\oplus}\{r\}} \frac{\mathrm{d}}{\mathrm{d}r} u_l + \left(U_{\mathrm{n}}\{r\} + \frac{\hbar^2 l(l+1)}{2m_{\mathrm{n}^\oplus}\{r\} r^2} \right) u_l = \mathcal{E} u_l$$

$$\dot{u}_l = \frac{\partial u_l}{\partial \mathcal{E}}$$

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A_{lm} and B_{lm} are fixed by matching ϕ and $\nabla\phi$ on the sphere.

Neutron band structure

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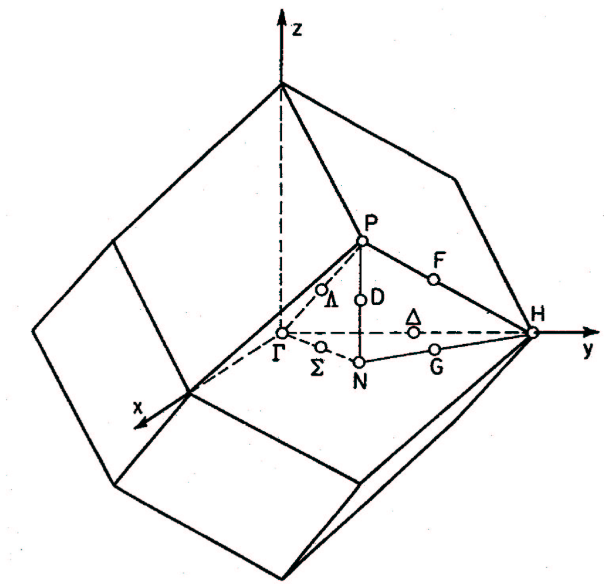
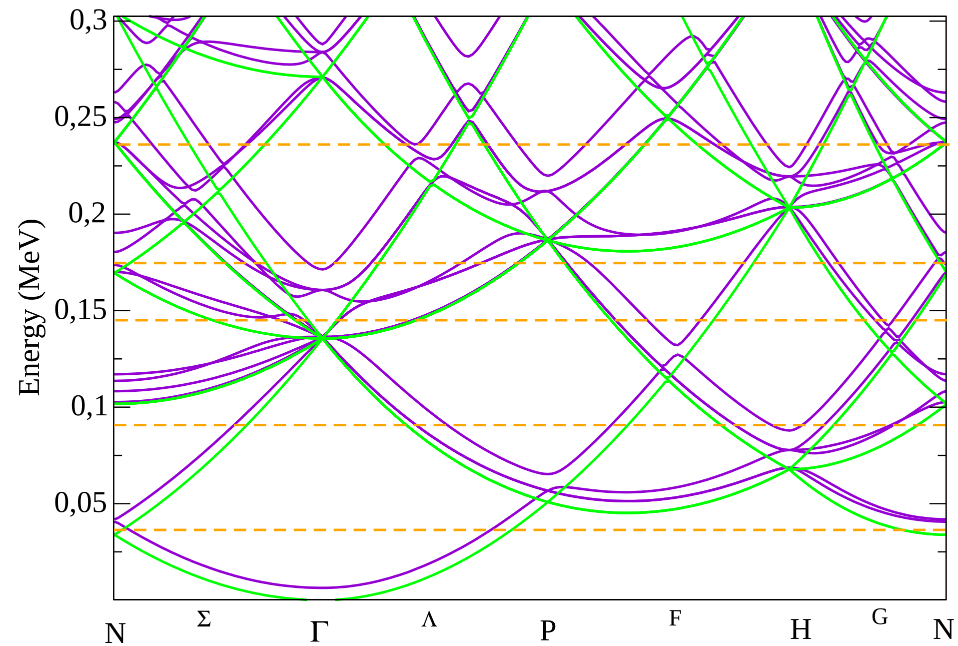
Band structure, W-S approximation with Negele & Vautherin boundary conditions, Fermi gas





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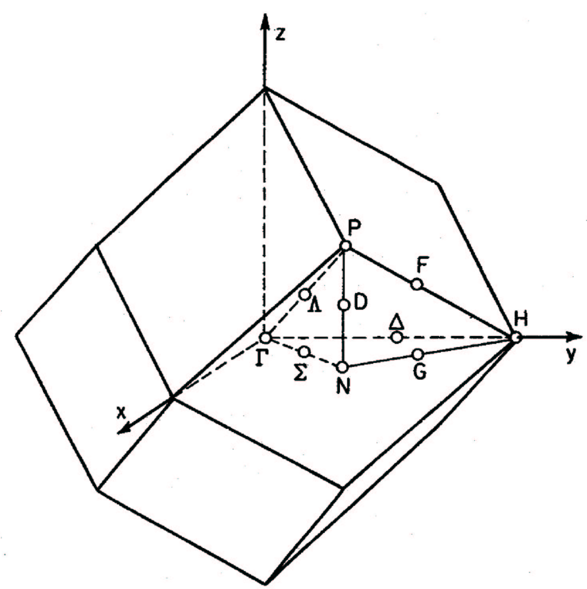

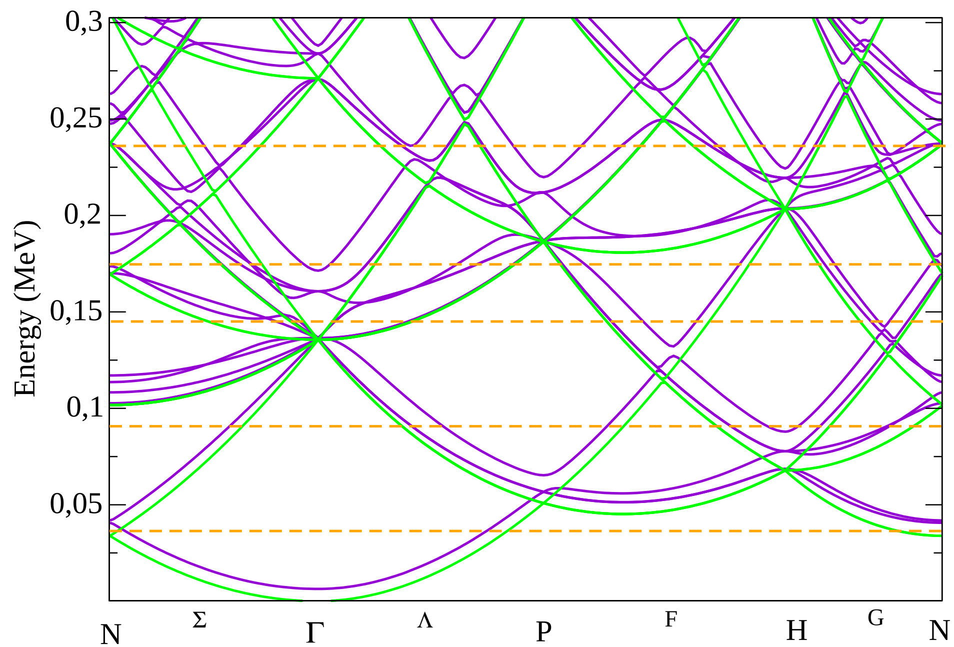

$\Rightarrow$ Despite strong nuclear potential, energy spectrum is very close to that of ideal Fermi gas except for avoided crossings

PK cancellation theorem

Phillips & Kleinman theorem, Phys. Rev. 116 (1959) 287.

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$\Rightarrow$ dripped neutrons in the bulk behave as nearly free particles !

Band gaps and “neutronics”

Lattice spacing, nuclear composition and density of dripped neutrons vary with depth

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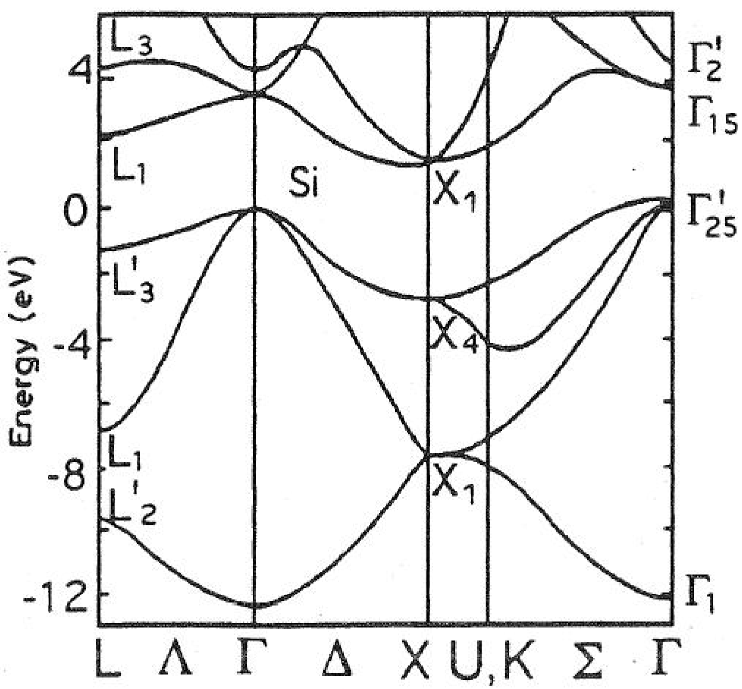

Does there exist a neutronic band gap in some layers of the inner crust ?

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Signatures of band gaps in the single particle density of states

$$\mathcal{N}\{\mathcal{E}\} = \frac{\mathrm{d}n}{\mathrm{d}\mathcal{E}} = \sum_{\alpha} \int_{\mathrm{BZ}} \frac{\mathrm{d}^3 k}{(2\pi)^3} \delta\{\mathcal{E} - \mathcal{E}_{\alpha}\{\mathbf{k}\}\}$$

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$$\Rightarrow \int f\{\mathbf{k}\}\mathrm{d}S_{_\mathrm{F}} \simeq \sum_c w_c f\{\mathbf{k_c}\}S_c$$

S_c can be calculated analytically

Density of states and band gaps

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band theory

$$\mathcal{N}\{\mathcal{E}\} = \frac{1}{(2\pi)^3\hbar} \oint \frac{\mathrm{d}S_\varepsilon}{v}$$

ideal Fermi gas

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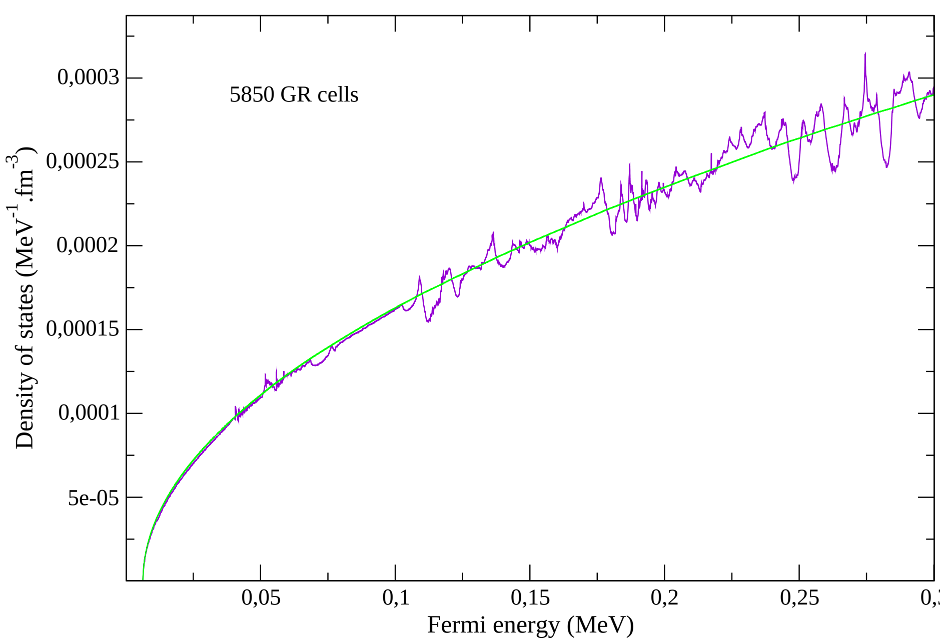

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Neutron current in the crust frame ?

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$$n^i = \sum_\alpha \int v^i_{\alpha k} \, \frac{\mathrm{d}^3 k}{(2\pi)^3} \, , \qquad\qquad v^i_{\alpha k} = \frac{1}{\hbar} \frac{\partial \mathcal{E}_\alpha}{\partial k_i}$$

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Lowest energy state for given density n_{n} and current n^i ?
$\Rightarrow$ uniform displacement of the Fermi surface

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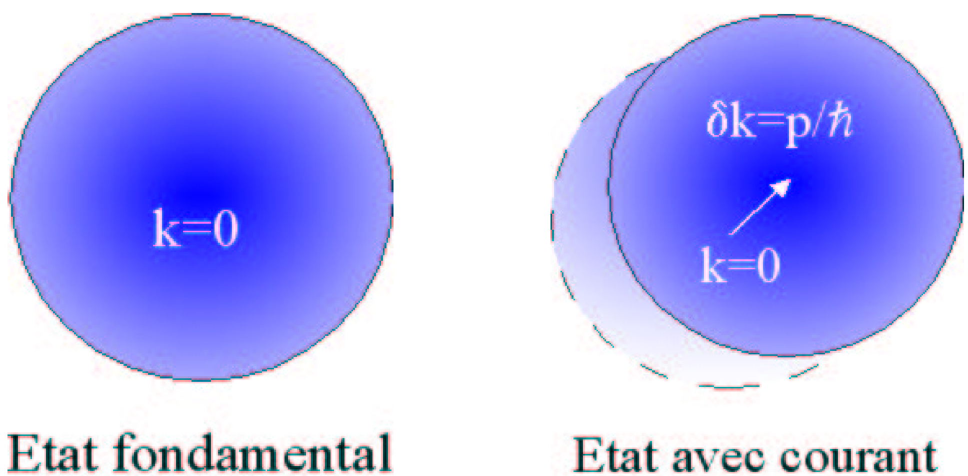

Neutron transport properties

Carter, Chamel, Haensel, Nucl. Phys. A748 (2005) 675.
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$\Rightarrow$ Neutron transport properties are determined by the shape of the Fermi surface

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Carter, Chamel $\&$ Haensel, astro-ph/0408083.
At macroscopic scales $\gg$ lattice spacing

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Ex : If $m_\star$ is sufficiently large $\Rightarrow$ superfluid two-stream instability
Andersson *et al*, MNRAS 354 (2004) 101

Homogeneous nuclear matter

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$$\mathcal{E}\{\mathbf{k}\} = \frac{\hbar^2 k^2}{2m_{\mathrm{n}}{}^{\oplus}} + U_{\mathrm{n}}$$

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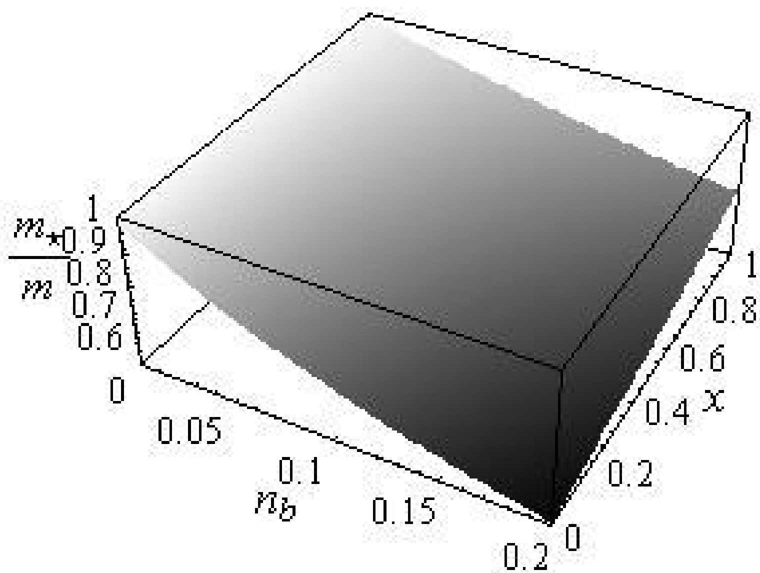

Neutron star crust matter

Different length scales !

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★ Microscopic scales $<$ nuclear size

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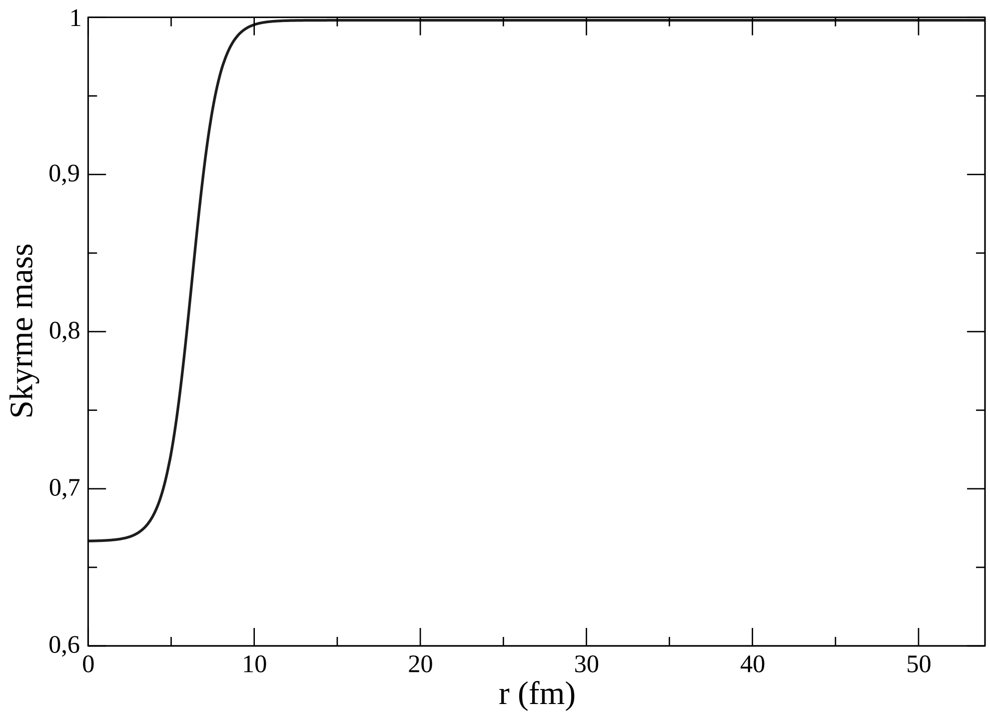

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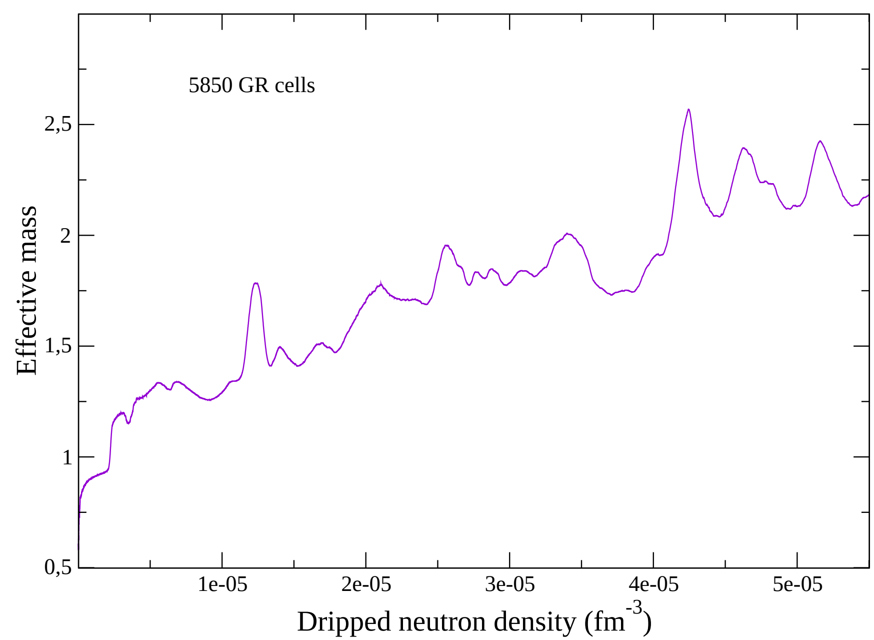

Metal or insulator ?

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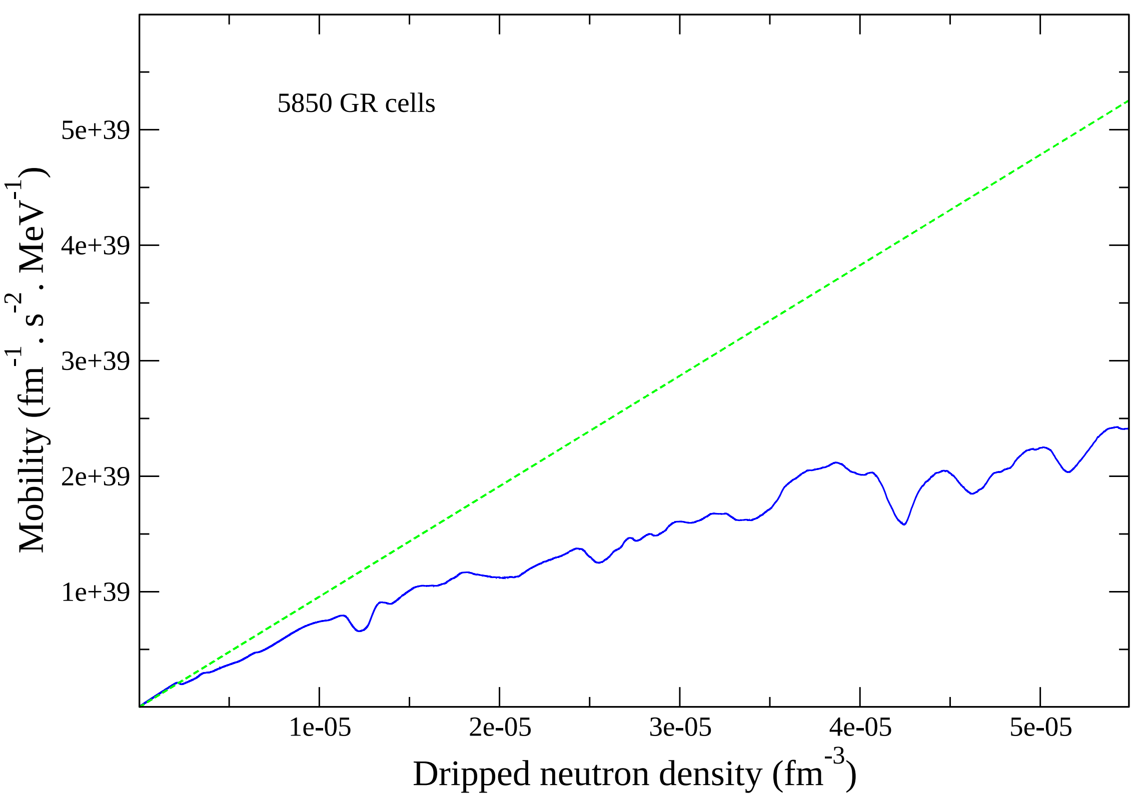

Bragg scattering and Fermi surface topology

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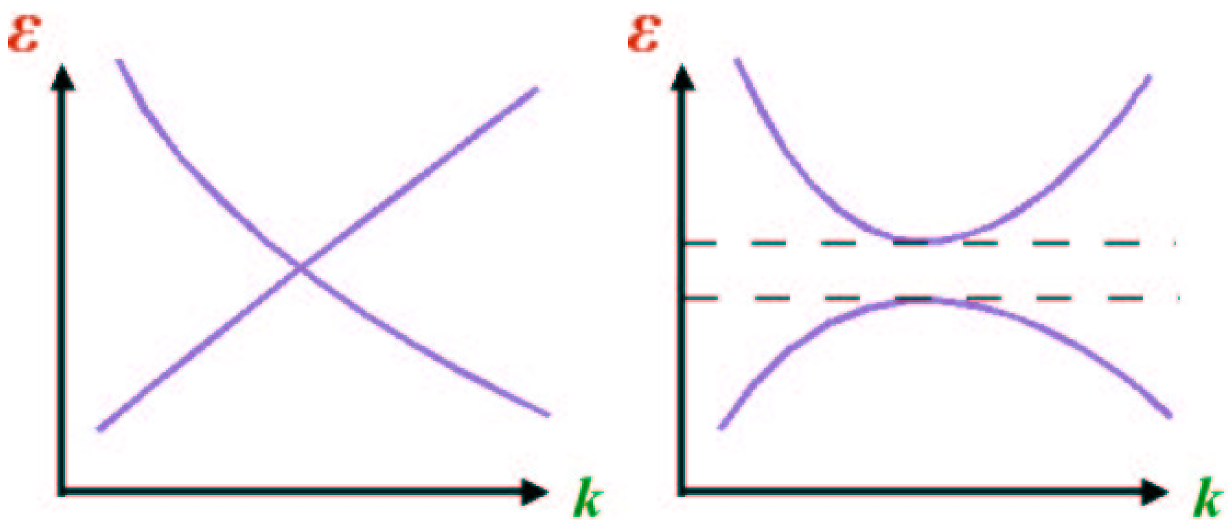
$\Rightarrow$ avoided crossings



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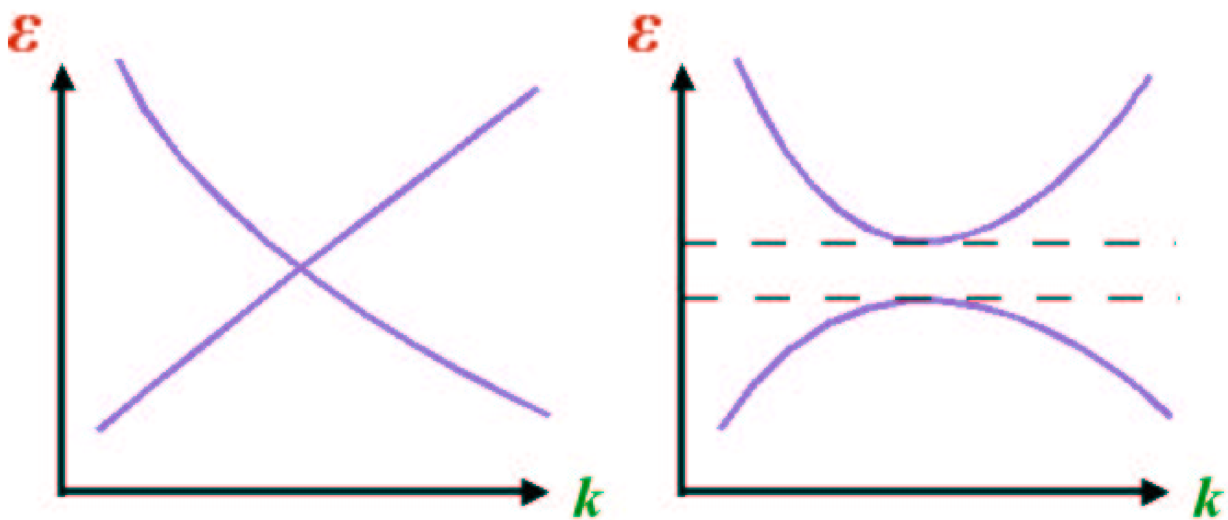
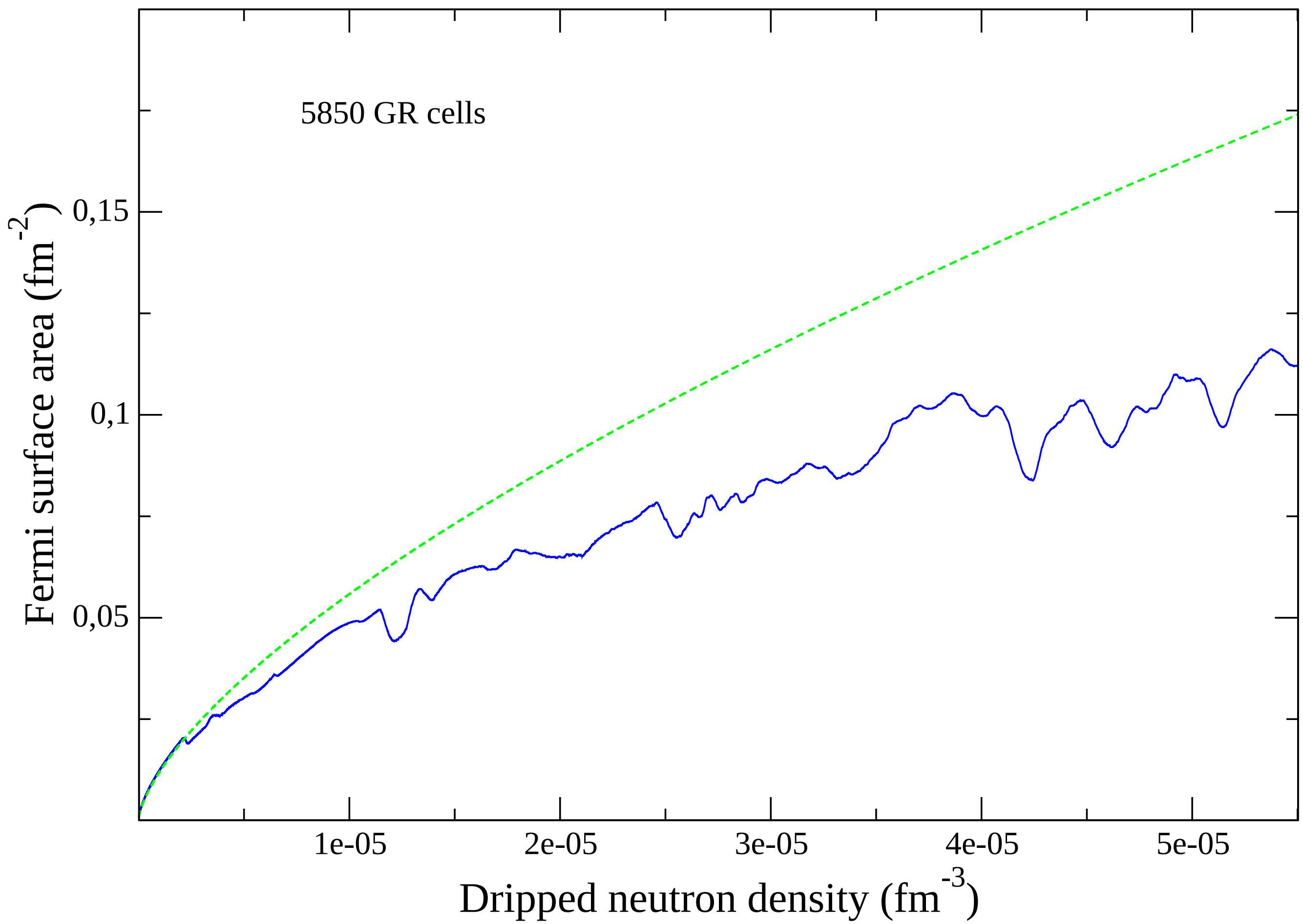


$\Rightarrow$ holes on the Fermi surface !

Fermi surface area



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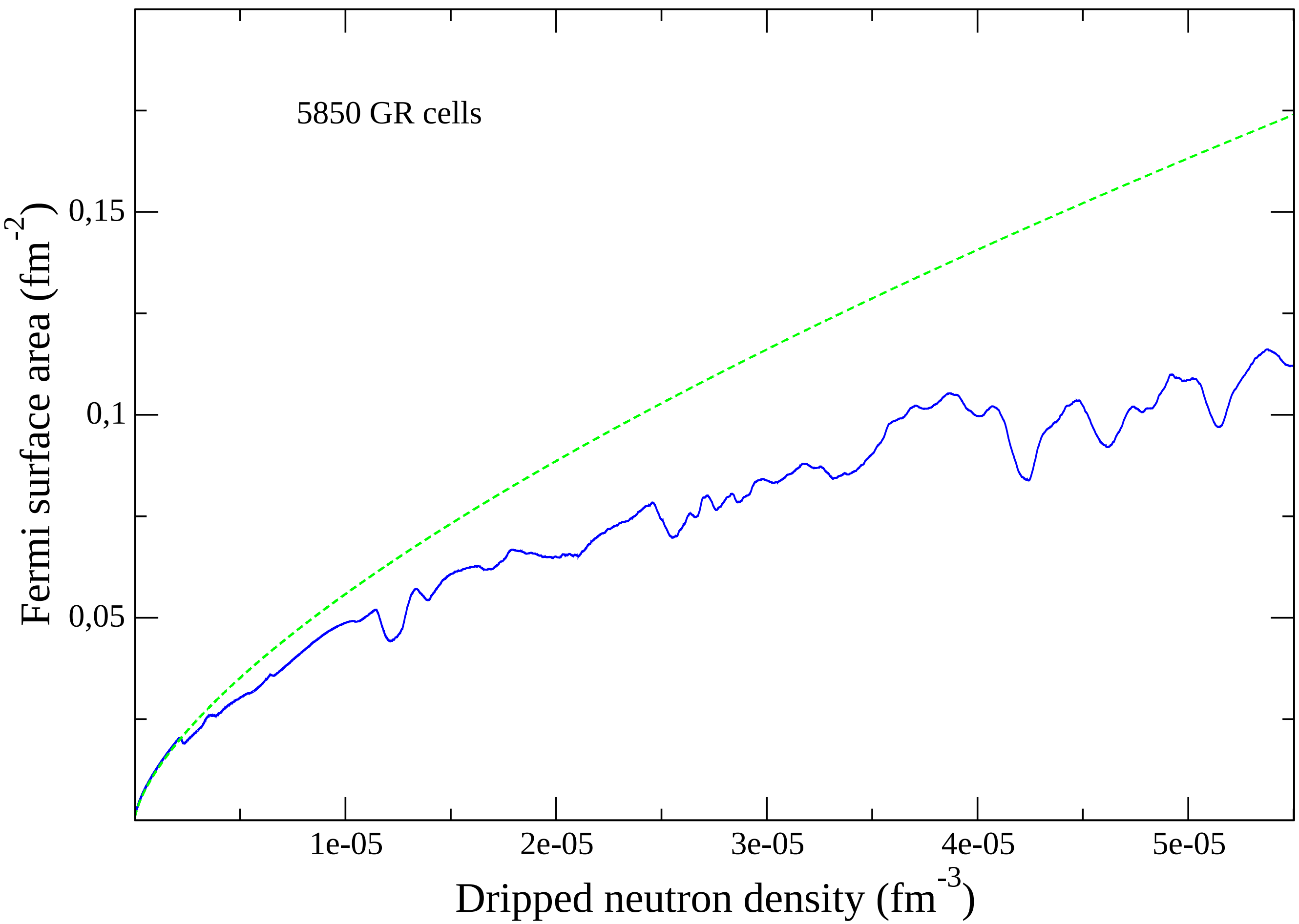

⇒ Fermi surface area is reduced while volume is unchanged

Fermi surface and effective mass

Effects of Bragg scattering ?

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★ around avoided crossings

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Effective mass is a probe of the topology of the Fermi surface

Deep layers of the inner crust

T-F calculation of Oyamatsu, Nucl. Phys. A561(1993) 431.

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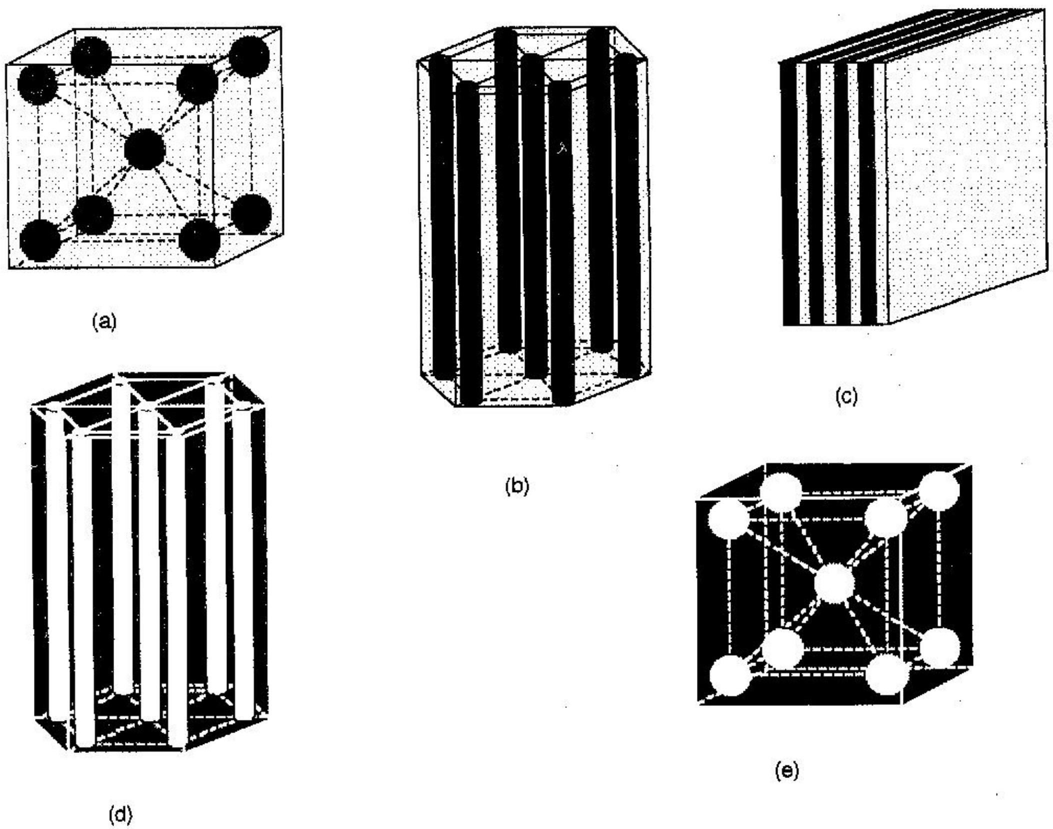

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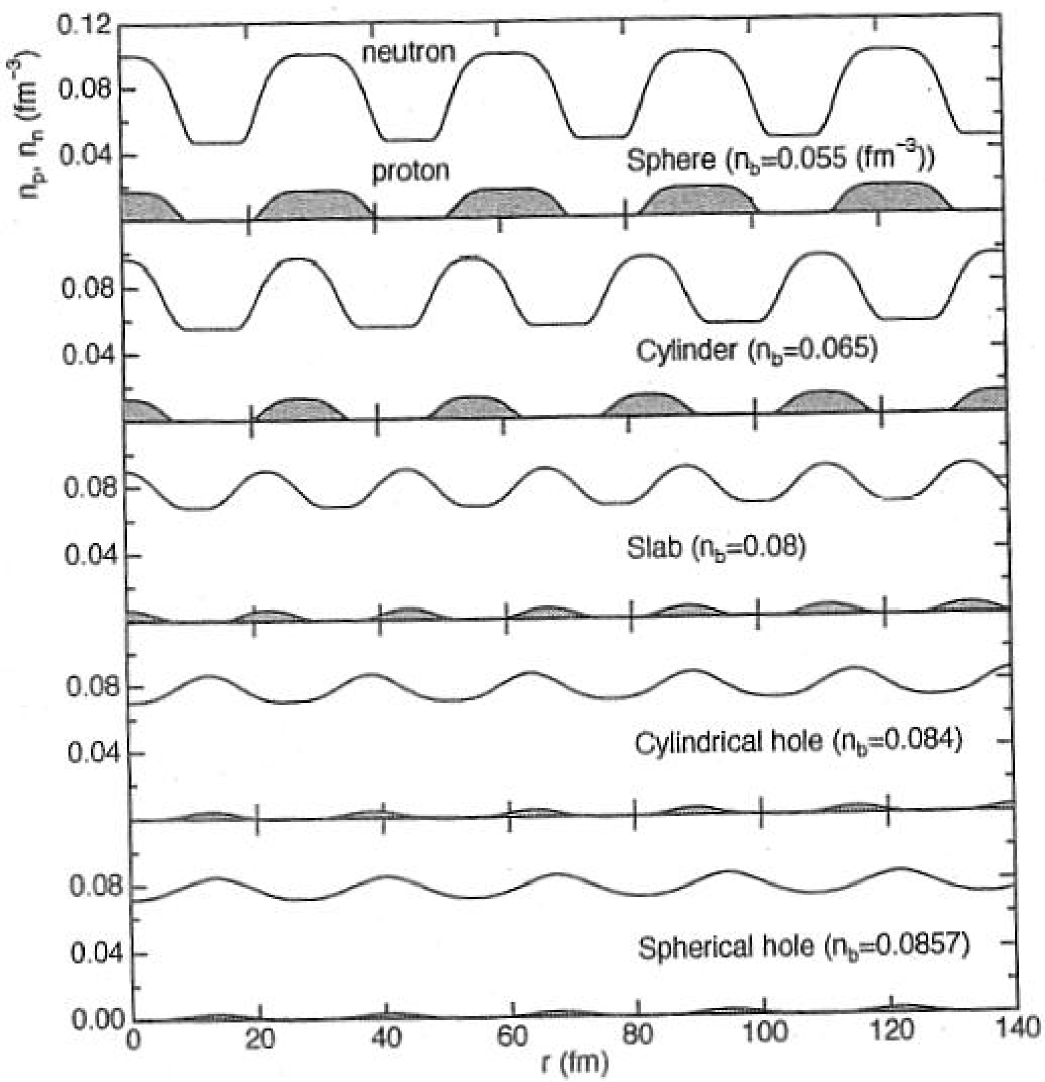
Thomas-Fermi calculation of Oyamatsu, Nucl. Phys. A561(1993) 431.



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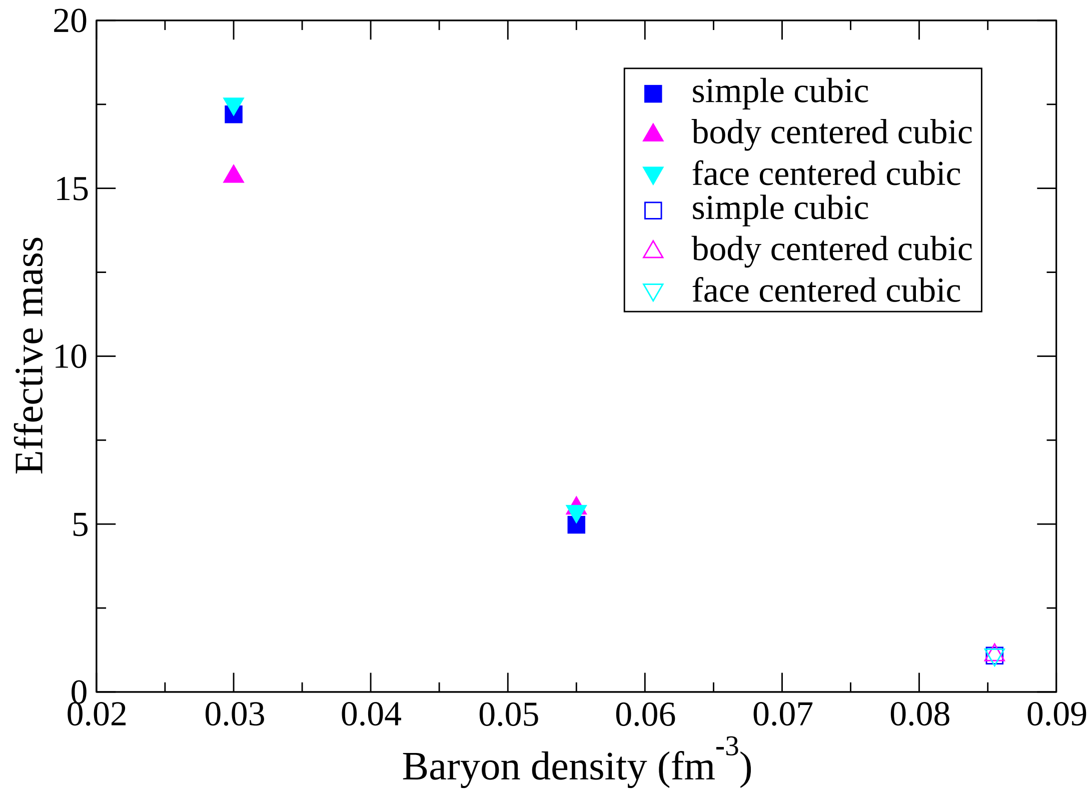
Equations are solved on a plane wave basis set

$$\varphi_{\mathbf{k}}\{\mathbf{r}\} = \sum_{\alpha} c_\alpha e^{{\rm i}\,(\mathbf{k}+\mathbf{K}_\alpha)\cdot\mathbf{r}}\,, \qquad \frac{\hbar^2(\mathbf{k}+\mathbf{K}_\alpha)^2}{2m_{\rm n}} < \mathcal{E}_{\rm cutoff}$$

Effective mass in layers of spherical nuclei/bubbles

Chamel, Nucl.Phys. A747 (2005) 109.



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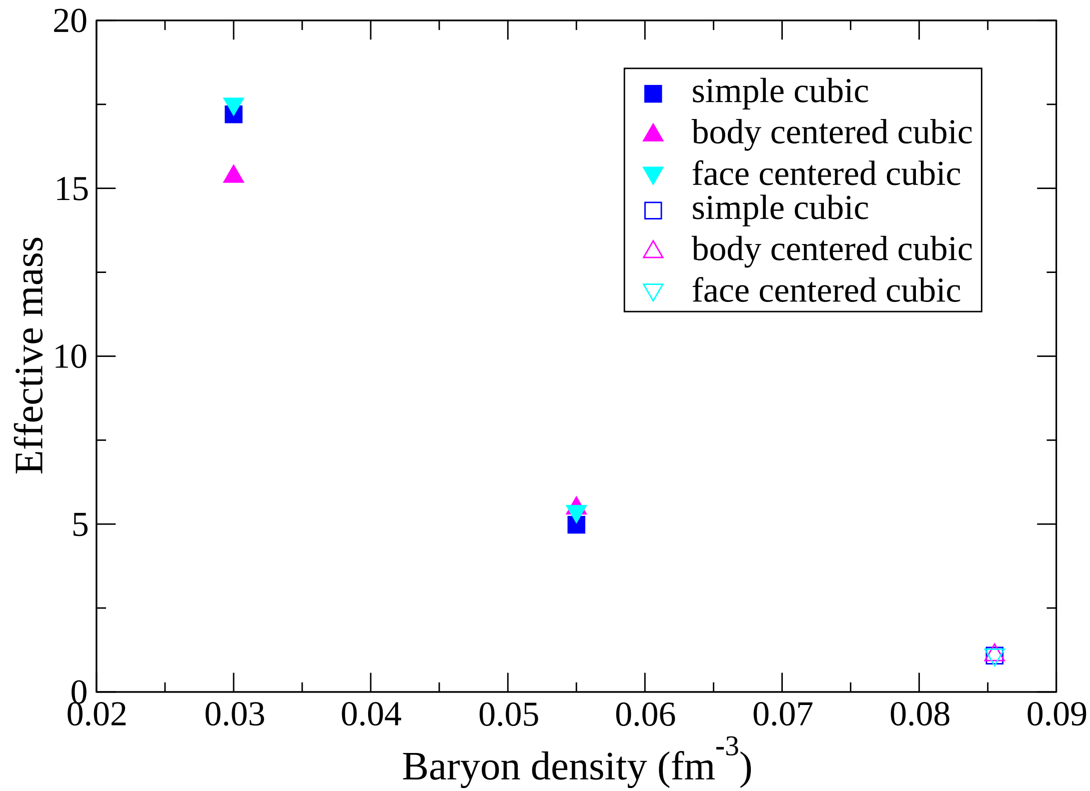

$\Rightarrow$ strong enhancement of $m_\star$ at low densities

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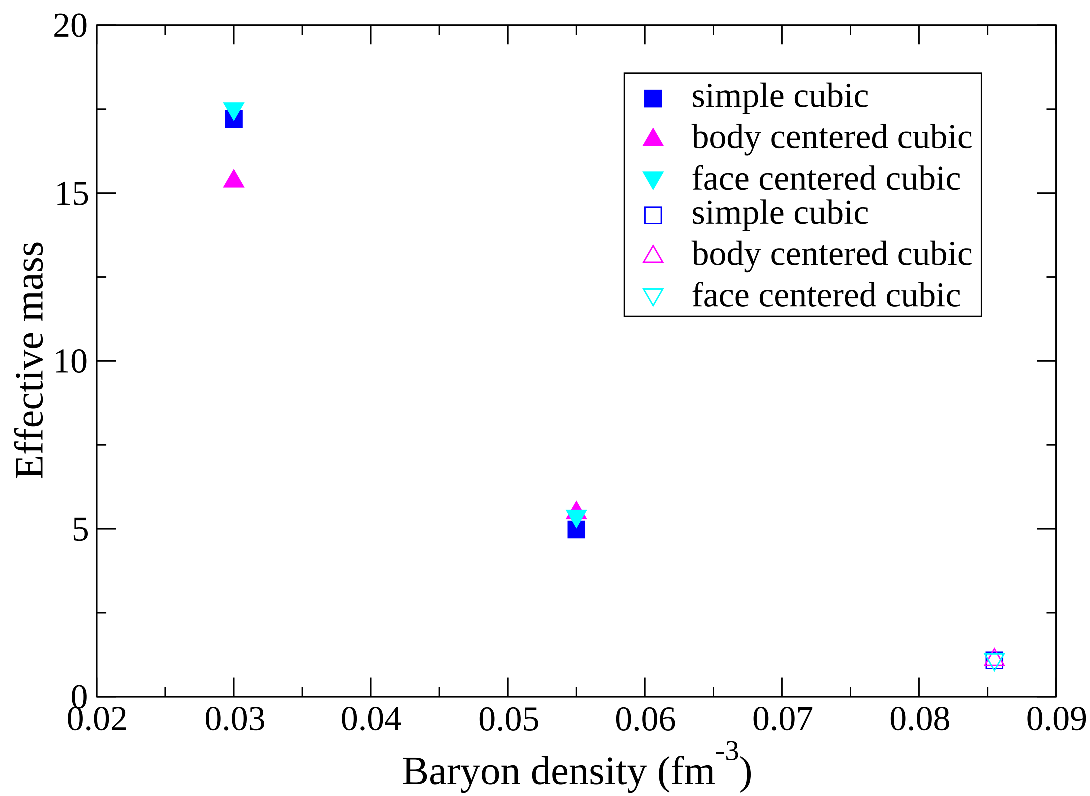

⇒ strong enhancement of $m_\star$ at low densities

⇒ small dependence on lattice structure

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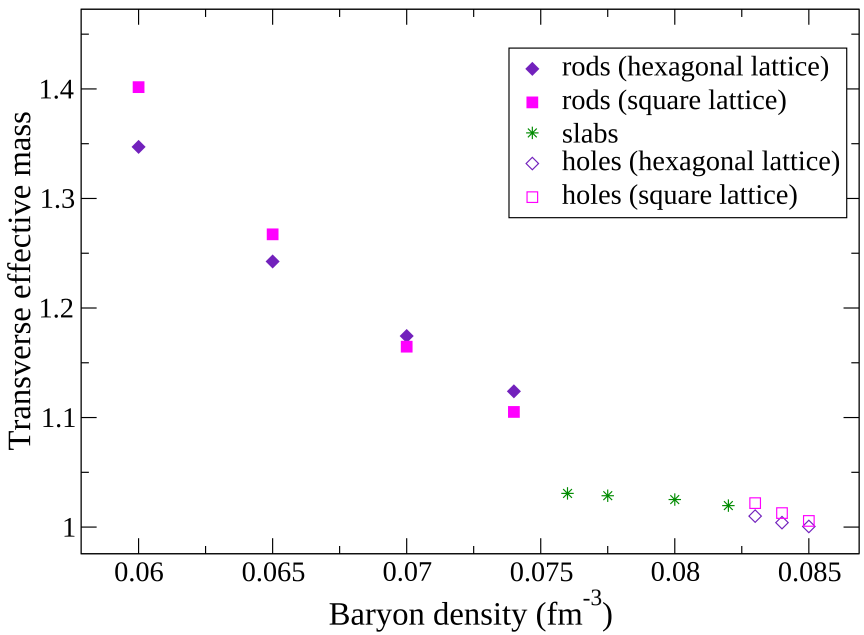

transverse effective mass
$m_\star{}^{\perp} \equiv n_{\rm f}/\mathcal{K}^{\perp}$

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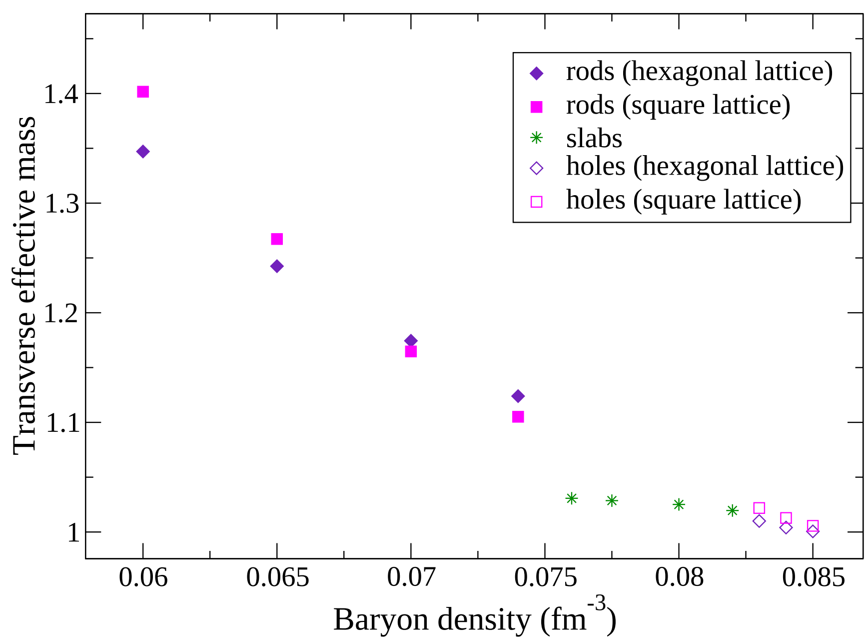

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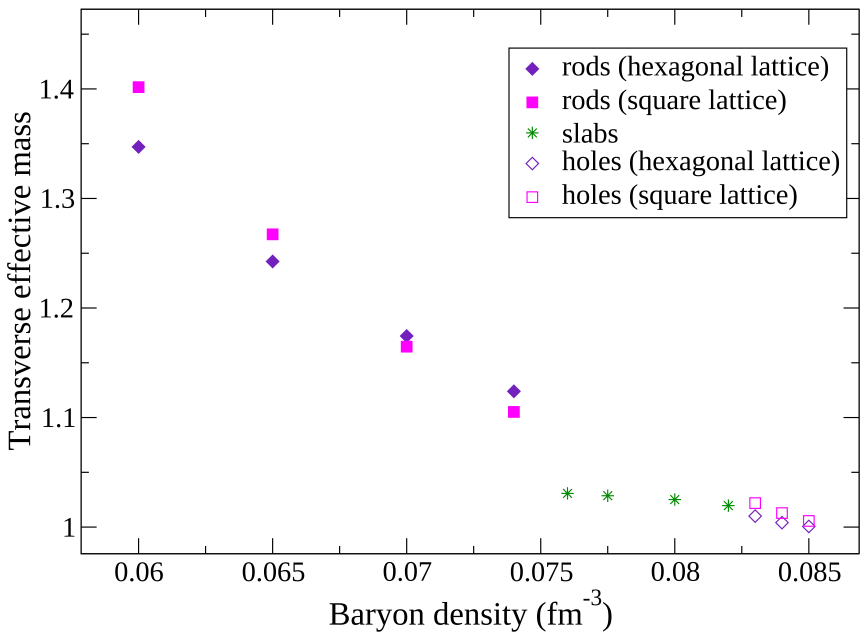

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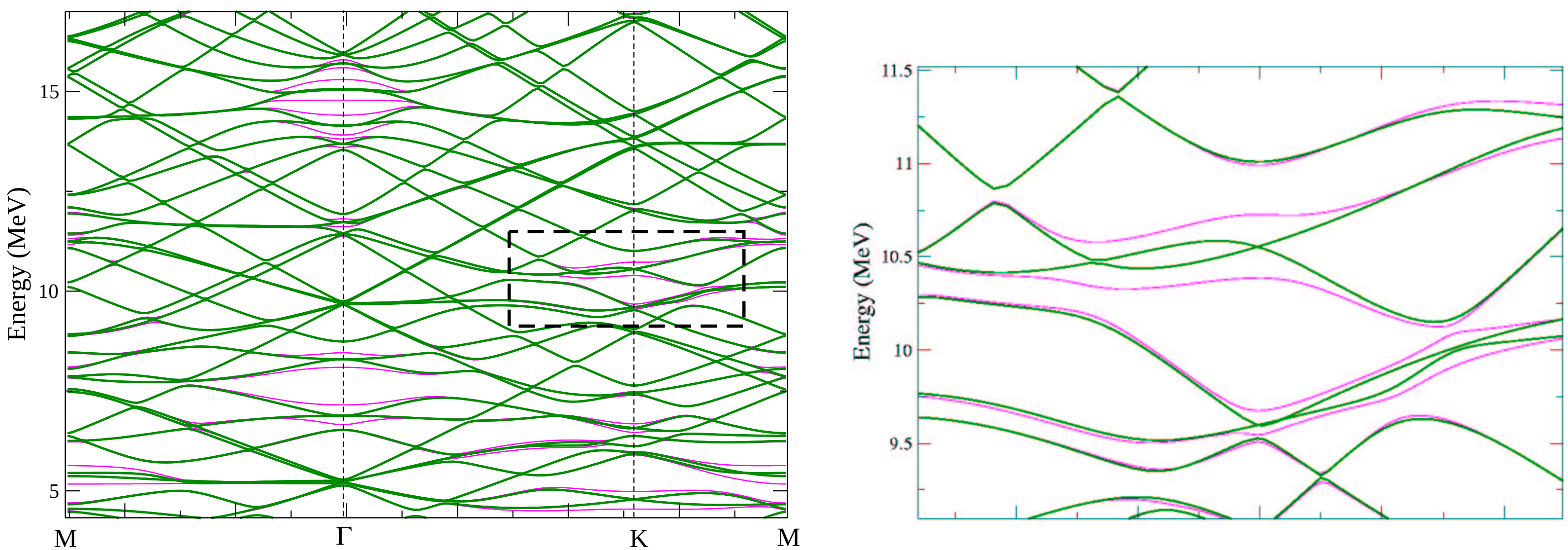
$\Rightarrow$ $m_\star \sim m_{\rm n}$ since pasta phases are nearly homogeneous

$\Rightarrow$ mobility is bounded $\mathcal{K} \geq 2n_{\rm n}/3m_{\rm n}$ (1D) and $\mathcal{K} \geq n_{\rm n}/3m_{\rm n}$ (2D)

Spin-orbit coupling

Chamel, Nucl.Phys. A747 (2005) 109.
Cylinder shaped nuclei ($n_{\rm b} = 0.06$ fm^{-3}) with spin-orbit coupling

$$V_{\rm LS}\{r\} = \frac{1}{r}\left(\lambda_1 \frac{{\rm d}n_{\rm b}}{{\rm d}r} - \lambda_2 \frac{\rm d}{{\rm d}r}(n_{\rm n} - n_{\rm p})\right)\frac{1}{2} l_z \sigma_z$$



$\Rightarrow m_\star{}^\perp/m_{\rm n} = 1.35 \mapsto m_\star{}^\perp/m_{\rm n} = 1.37$

Superfluidity and Bragg scattering

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★ Mean field approximation

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$\Rightarrow$ Hartree-Fock Bogoliubov (Bogoliubov-de Gennes) equations

$$\begin{pmatrix} \mathcal{H}\{\mathbf{r}\} - \mu & \Delta\{\mathbf{r}\} \\ \Delta^*\{\mathbf{r}\} & -\mathcal{H}^*\{\mathbf{r}\} + \mu \end{pmatrix} \begin{pmatrix} u\{\mathbf{r}\} \\ v\{\mathbf{r}\} \end{pmatrix} = E \begin{pmatrix} u\{\mathbf{r}\} \\ v\{\mathbf{r}\} \end{pmatrix}$$

assuming contact interactions in particle-particle and particle-hole channels

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★ Lattice symmetry $\Delta\{\mathbf{r}+\mathbf{T}\} = \Delta\{\mathbf{r}\}$ and $\mathcal{H}\{\mathbf{r}+\mathbf{T}\} = \mathcal{H}\{\mathbf{r}\}$

Superfluidity and Bragg scattering

★ Mean field approximation

$\Rightarrow$ Hartree-Fock Bogoliubov (Bogoliubov-de Gennes) equations

$$\begin{pmatrix} \mathcal{H}\{\mathbf{r}\} - \mu & \Delta\{\mathbf{r}\} \\ \Delta^*\{\mathbf{r}\} & -\mathcal{H}^*\{\mathbf{r}\} + \mu \end{pmatrix} \begin{pmatrix} u\{\mathbf{r}\} \\ v\{\mathbf{r}\} \end{pmatrix} = E \begin{pmatrix} u\{\mathbf{r}\} \\ v\{\mathbf{r}\} \end{pmatrix}$$

assuming contact interactions in particle-particle and particle-hole channels

★ Lattice symmetry $\Delta\{\mathbf{r}+\mathbf{T}\} = \Delta\{\mathbf{r}\}$ and $\mathcal{H}\{\mathbf{r}+\mathbf{T}\} = \mathcal{H}\{\mathbf{r}\}$

$\Rightarrow$ Floquet-Bloch theorem

$$u_{\mathbf{k}}\{\mathbf{r}+\mathbf{T}\} = e^{\mathrm{i}\,\mathbf{k}\cdot\mathbf{T}} u_{\mathbf{k}}\{\mathbf{r}\} \quad v_{\mathbf{k}}\{\mathbf{r}+\mathbf{T}\} = e^{\mathrm{i}\,\mathbf{k}\cdot\mathbf{T}} v_{\mathbf{k}}\{\mathbf{r}\}$$

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$\Delta\{\mathbf{r}\}$ slowly varying

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Pairing and effective mass

Carter, Chamel, Haensel (2005), Nucl. Phys. A in press
Effects of pairing on mobility ?

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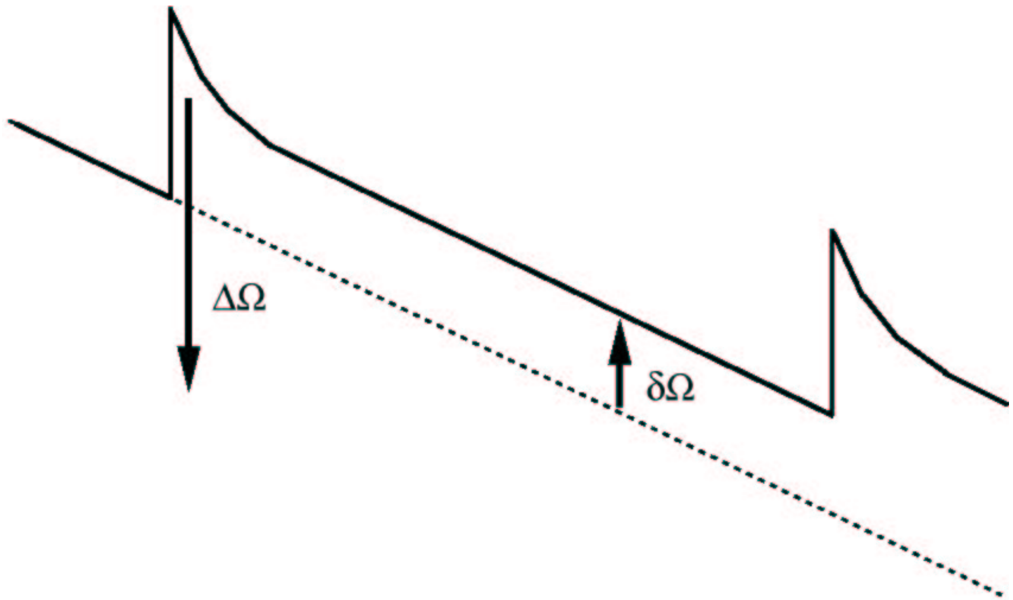
In uniform nuclear matter $m_\star \equiv n_\mathrm{n}/\mathcal{K} = m_\mathrm{n}{}^\oplus$ independent of Δ

Neutronics in the crust and pulsar glitches

Neutronics in the crust and pulsar glitches

★ Some pulsars suddenly spin up $\Rightarrow$ glitches $\delta\Omega > 0$



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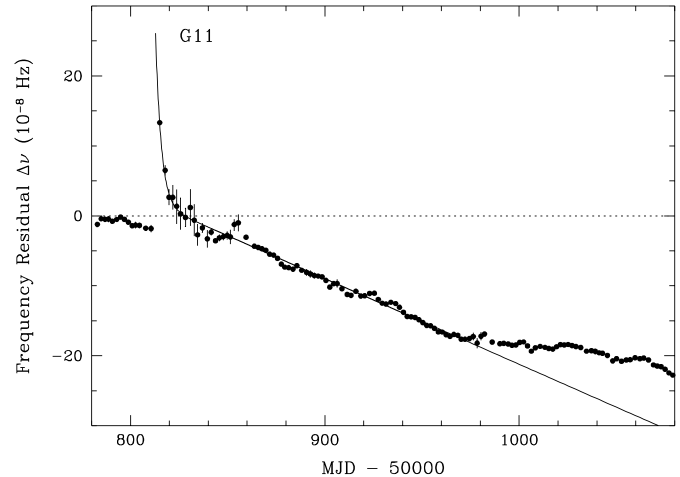

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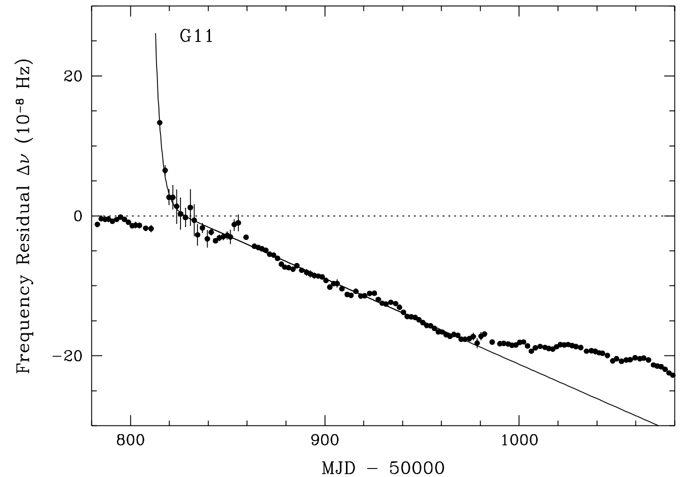

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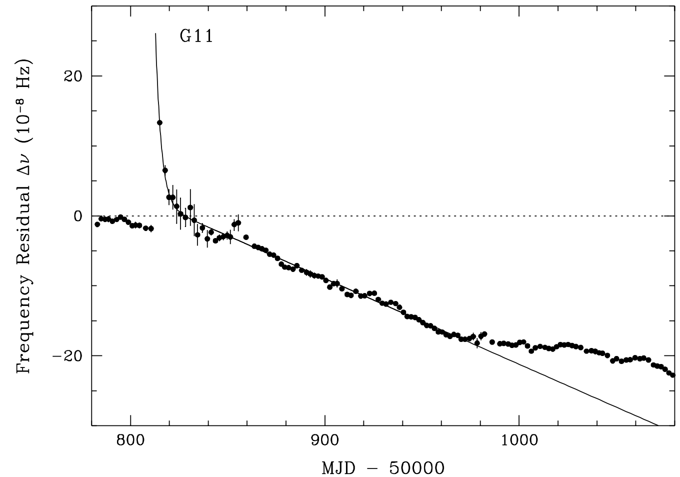

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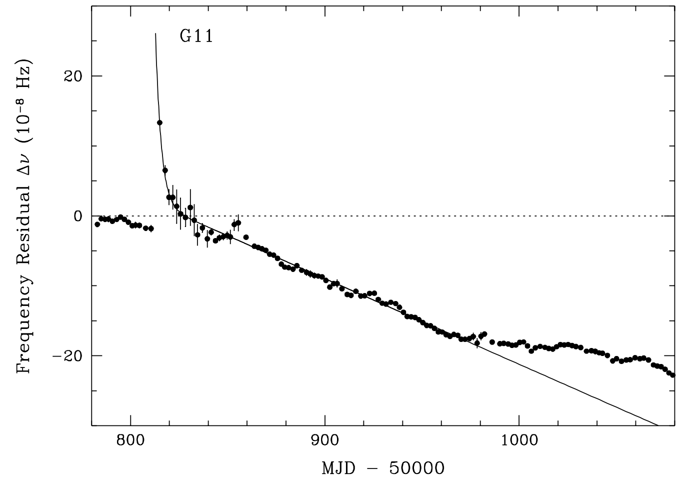
★ Some pulsars suddenly spin up ⇒ glitches $\delta\Omega > 0$



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★ Neutron transport in the crust is strongly affected by Bragg scattering ($m_\star \gg m_{\rm n}$)

⇒ glitches are direct probes for neutronics in the crust !

Summary & Perspectives

- ★ Structure of the outer crust completely determined by known nuclei up to $\rho \sim 10^{11}$ g.cm^{-3}
- ★ So far only one fully self-consistent quantum calculation of the inner crust by Negele& Vautherin (1973) $\Rightarrow$ strong proton shell effects, size of nuclear clusters $\sim$ independent of density. Consistent results from TF and CLDM.
- ★ But recent calculations of Baldo *et al.* $\Rightarrow$ no shell effects, Z very sensitive to pairing ! Remain to be clarified. Medium effects on the pairing field ? Superfluidity in the crust $\Rightarrow$ cooling, pulsar glitches.
- ★ Necessity to go beyond the W-S approximation to study neutron transport in the crust $\Rightarrow$ band theory. Bragg scattering $\Rightarrow$ strong enhancement of neutron mass ! Neutronics in the crust $\Rightarrow$ oscillations modes, pulsar glitches
- ★ Neutron band effects on the structure of the crust ? on the superfluidity ? Effects of disorders (impurities, defects, *etc.*) on the transport properties ?

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