Superfluid models of neutron stars

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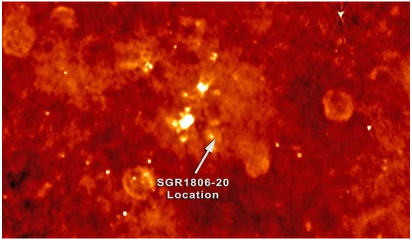
Motivations

Neutron stars are not static, they evolve and can undergo instabilities triggered by

- spin-down
- thermonuclear explosions
- starquakes
- magnetic field...



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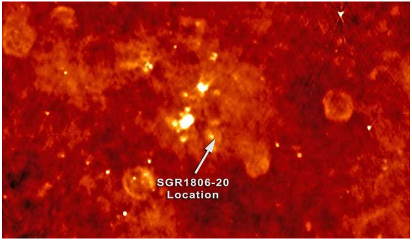
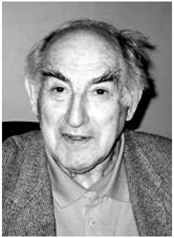


Punchlines

In order to understand the dynamical evolution of neutron stars, one needs to construct global models which take into account the internal composition of the star. Besides all the microscopic coefficients have to be calculated consistently.

Superfluidity in neutron stars

Theoretically, superfluidity of neutron star matter is well established.



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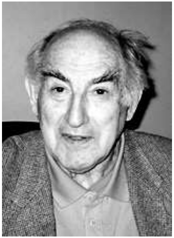

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but $T_{cn}(\rho)$ and $T_{cp}(\rho)$ are very uncertain

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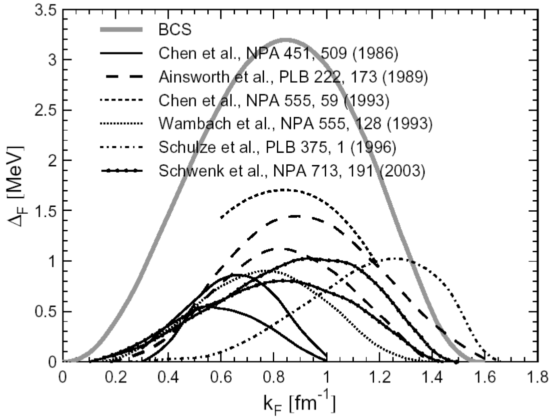
Superfluidity in neutron stars

Example : $^1S_0$ pairing gap in uniform neutron matter

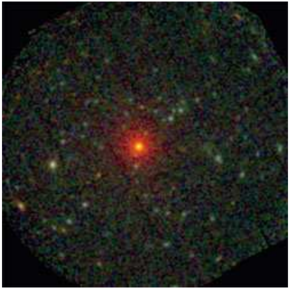


Lombardo & Schulze, Lect. Notes Phys. 578 (2001) Springer

Superfluidity in neutron stars

Observational evidence of superfluidity ?

- pulsar glitches (long relaxation times, vortex pinning scenario)
- neutron star thermal X-ray emission (cooling)



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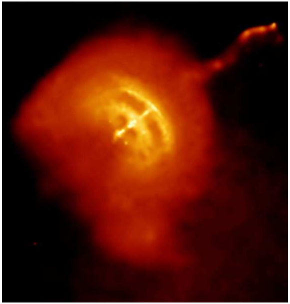
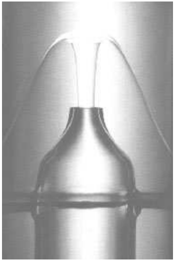


Vela pulsar

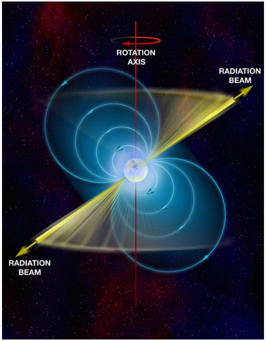
Superfluids are multi-fluid systems



One of the striking consequences of superfluidity is the possibility of having distinct dynamical components inside the fluid.

Example : many properties of superfluid helium can be explained by a two-fluid model

How many fluids inside neutron stars ?



Unlike neutrons, electrically charged particles are essentially locked together by the interior magnetic field on very long timescales of the order of the age of the star

Easson, ApJ 233(1979), 711

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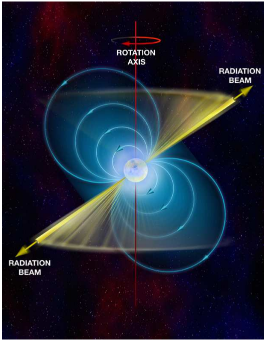

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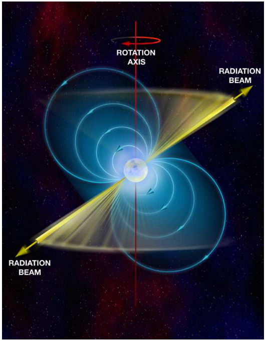

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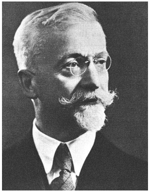
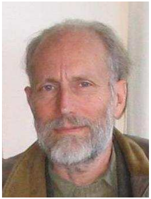
mutual entrainment

Due to the strong interactions between neutrons and protons, the two fluids are coupled by Andreev&Bashkin effects : momentum and velocity are not aligned

How to obtain flow equations for fluid mixtures ?





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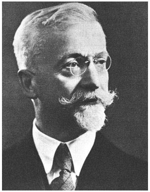

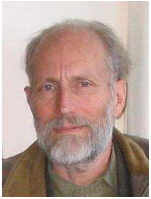

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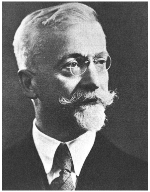

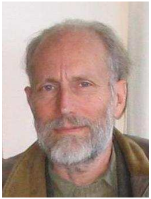

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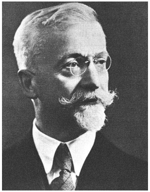

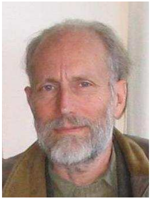

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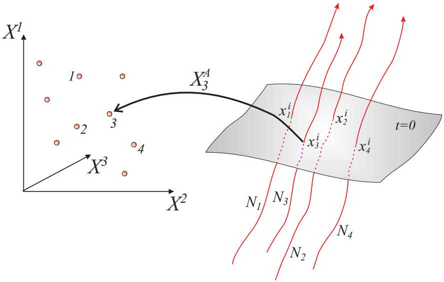
Action principle $$\int \Lambda\{n_{\rm X}^{\mu}\}\, \mathrm{d}^4 x$$

The Lagrangian density or master function Λ depends on the 4-current vectors $n_{\rm X}^{\mu} = n_{\rm X} u_{\rm X}^{\mu}$ of the different fluids X

Variational formulation of hydrodynamics



picture from Andersson&Comer

Consider variations of the fluid particle trajectories

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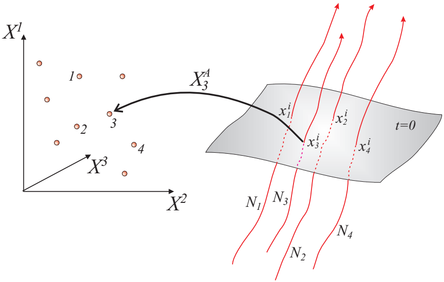

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$$\Rightarrow n_x^\mu \varpi_{\mu\nu}^x + \pi_\nu^x \nabla_\mu n_x^\mu = f_\nu^x$$

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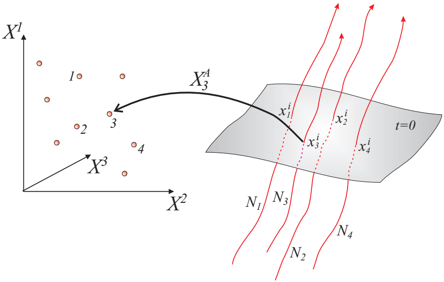

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Consider variations of the fluid particle trajectories

$$\Rightarrow n^{\mu}_{\mathrm{x}} \varpi^{\mathrm{x}}_{\mu\nu} + \pi^{\mathrm{x}}_{\nu} \nabla_{\mu} n^{\mu}_{\mathrm{x}} = f^{\mathrm{x}}_{\nu}$$

4-momentum covector

$$\pi^{\mathrm{x}}_{\mu} = \frac{\partial \Lambda}{\partial n^{\mu}_{\mathrm{x}}}$$

vorticity 2-form

$$\varpi^{\mathrm{x}}_{\mu\nu} = 2\nabla_{[\mu} \pi^{\mathrm{x}}_{\nu]} = \nabla_{\mu} \pi^{\mathrm{x}}_{\nu} - \nabla_{\nu} \pi^{\mathrm{x}}_{\mu}$$

4-force density covector

$$f^{\mathrm{x}}_{\nu}$$

Variational formulation of hydrodynamics

Stress-energy density tensor of the fluids can be obtained from Noether theorem

$$T^{\mu}_{\ \nu} = \Psi\, \delta^{\mu}_{\nu} + \sum_{\mathrm{x}} n^{\mu}_{\mathrm{x}} \pi^{\mathrm{x}}_{\nu}$$

where Ψ is a generalized pressure

$$\Psi = \Lambda - \sum_{\mathrm{x}} n^{\mu}_{\mathrm{x}} \pi^{\mathrm{x}}_{\mu} \ .$$

In general Ψ depends on the velocities of the fluids.

Note that the above expressions are valid for both Newtonian and relativistic fluids.

Superfluid models of neutron stars

Ultimately realistic models of neutron stars should describe three distinct regions, which can be treated within the same variational formalism :

- **the outer crust**

  Carter, Chachoua, Chamel, Gen.Rel.Grav.38 (2006)83.
- **the inner crust**

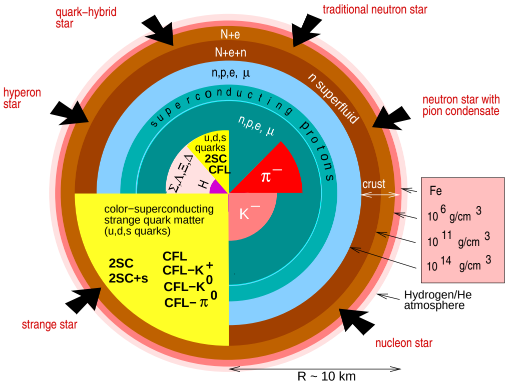
  Carter&Samuelsson, Class. Quant. Grav. 23 (2006)5367.
  (simplified treatment based on a two-fluid model but including stratification :
  Carter, Chamel, Haensel, P., Int.J.Mod.Phys.D15(2006)777.
  Chamel, Carter, MNRAS 368(2006)796.)
- **the liquid core**

The different layers have to be matched with appropriate boundary conditions.

Superfluid model of neutron star core

Main assumptions :

- $T = 0$, $npe\mu$ composition ($\rho_0/2 \lesssim \rho \lesssim 3\rho_0$)
- charged particles are comoving due to magnetic field
- superfluid neutrons can move with a different velocity



picture from F. Weber

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It is formally straightforward to include magnetic field in the action principle, even in the Newtonian context

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But this requires a much better understanding of superconductivity and dynamics of charged particles.

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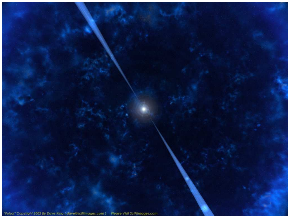

At the microscopic scale, nucleons are essentially non-relativistic at densities below $\sim 3\rho_0$.

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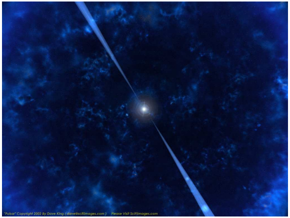

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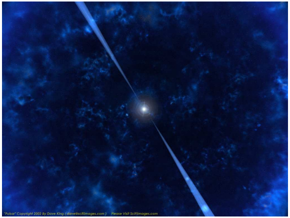

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But use a 4D covariant approach to then extend to GR

Carter & Chamel,Int.J.Mod.Phys.D13 (2004), 291-326.

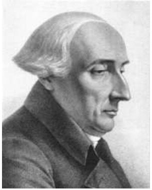
Carter & Chamel,Int.J.Mod.Phys.D14 (2005) 717-748.

Carter & Chamel,Int.J.Mod.Phys.D14 (2005) 749-774.

Non-relativistic Lagrangian density

Specify the Lagrangian density

$$\Lambda = \Lambda_{\text{mat}} + \Lambda_{\text{grf}}$$



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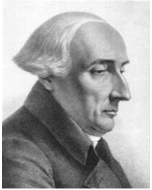
assuming *small* relative currents

$$\Rightarrow \Lambda_{\text{mat}} = \Lambda_{\text{dyn}} + \Lambda_{\text{ins}}$$



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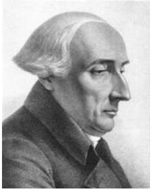

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$$\Lambda_{\rm dyn} = \frac{1}{2} \sum_{q,q'=n,p} \eta_{\mu\nu}\, \mathcal{K}^{qq'} n_q^\mu n_{q'}^\nu$$

dynamical term
(neglect lepton contribution)

$\mathcal{K}^{qq'}$ is a symmetric “mobility” matrix

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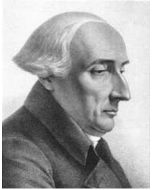

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Galilean invariance

$$\sum_{q'} n_{q'} \mathcal{K}^{qq'} = m$$

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$$\Lambda_{\rm ins} = -U_{\rm ins} - \rho\phi$$ internal static term

$U_{\rm ins}$ internal non-gravitational energy density
ϕ gravitational potential

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$$\Lambda_{\text{grf}} = -\frac{1}{8\pi G}\eta^{\mu\nu}(\nabla_\mu\phi)(\nabla_\nu\phi)$$

gravitational field term

LaTeX

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Variations with respect to ϕ leads to Poisson's equation

$$\eta^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 4\pi G\rho$$

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From non-relativistic to relativistic fluids

First include the effects of gravitation à la Cartan into the space-time

$$\gamma_{\mu\nu} = \eta_{\mu\nu} - 2\phi t_\mu t_\nu$$

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then remember that for weak gravitational fields, the Riemannian metric $g_{\mu\nu}$ of the relativistic space-time can be locally approximated by

$$g_{\mu\nu} \simeq \eta_{\mu\nu} - (c^2 + 2\phi) t_\mu t_\nu = \gamma_{\mu\nu} - c^2 t_\mu t_\nu$$

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Relativistic hydrodynamics

Relativistic Lagrangian density

$$\widetilde{\Lambda}_{\rm mat} = \sum_{k=0}^{+\infty} \lambda_k (x^2 - n_n n_p)^k$$

$$x^2 c^2 = -g_{\mu\nu} n_n^\mu n_p^\nu$$

$$n_n^2 c^2 = -g_{\mu\nu} n_n^\mu n_n^\nu$$

$$n_p^2 c^2 = -g_{\mu\nu} n_p^\mu n_p^\nu$$

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Add Einstein-Hilbert term to get Einstein's equations

$$\Lambda_{\mathrm{EH}} = \frac{c^4}{16\pi G} R$$

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Momentum vs velocity

From the Lagrangian density, we can obtain the 4-momentum covector ($q = n, p$)

$$\pi^q_\mu = \sum_{q'=n,p} g_{\mu\nu} \widetilde{\mathcal{K}}^{qq'} n^\nu_{q'}$$

Introduce relativistic effective masses

$$\widetilde{m}^q_\star \equiv n_q \widetilde{\mathcal{K}}^{qq}$$

Likewise one can introduce non-relativistic effective masses $m^q_\star$

$$\frac{\widetilde{m}^q_\star}{m} = \frac{m^q_\star}{m} + \frac{\mu_q}{mc^2} - 1 - \frac{1}{m} \frac{\partial \widetilde{\mathcal{K}}^{np}}{\partial n_q} \left(n_n n_p - x^2 \right)$$

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Evaluation of the microscopic parameters

Microscopic input parameters : $\lambda_k \; \{n_n, n_p, n_e, n_\mu\}$

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$\Rightarrow$ simple analytic expressions easy to implement numerically

Chamel&Haensel, Phys.Rev.C 73(2006), 045802

LaTeX

Superfluidity condition : microscopic scale

How to account for superfluidity ?

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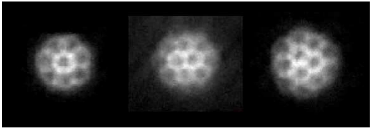

LKB, Ecole Norm. Sup., France

$$d_v \simeq 3.4 \times 10^{-3} \sqrt{\frac{10^2\,\mathrm{s}^{-1}}{\Omega}}\,\mathrm{cm}$$

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Superfluidity condition : microscopic scale

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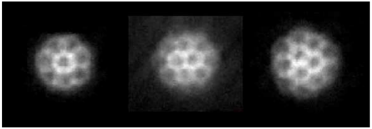

LKB, Ecole Norm. Sup., France

$$d_v \simeq 3.4 \times 10^{-3} \sqrt{\frac{10^2\,\mathrm{s}^{-1}}{\Omega}}\,\mathrm{cm}$$

Locally $\varpi^n_{\mu\nu} = 0 \Rightarrow \pi^n_\mu = (\hbar/2)\nabla_\mu \phi$ where ϕ is the quantum phase of the condensate

Superfluidity condition : mesoscopic scale

At scales $\gg d_v$, the superfluid mimics rigid body rotation

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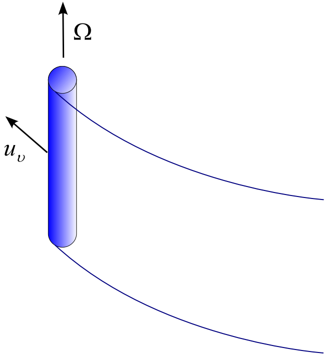

vorticity is carried along u^{μ}_{υ}

$$\Rightarrow u^{\mu}_{\upsilon} \varpi^{n}_{\mu\nu} = 0$$

Composition of neutron star core

The composition is determined by the rates of transfusion processes ($N = \varnothing, n, p$, $X = n, p, e, \mu$)

$$n + N \to p^+ + \ell + \bar{\nu}_\ell \,, \qquad p^+ + N + \ell \to n + \nu_\ell$$

$$e^- + X \to \mu^- + X + \bar{\nu}_\mu + \nu_e \,, \qquad \mu^- + X \to e^- + X + \bar{\nu}_e + \nu_\mu$$

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$$n + N \to p^{+} + \ell + \bar{\nu}_{\ell}\,, \qquad p^{+} + N + \ell \to n + \nu_{\ell}$$

$$e^{-} + X \to \mu^{-} + X + \bar{\nu}_{\mu} + \nu_{e}\,, \qquad \mu^{-} + X \to e^{-} + X + \bar{\nu}_{e} + \nu_{\mu}$$

- relaxation time $\ll$ time scales of interest
  $\Rightarrow$ equilibrium $\mathcal{A}^{\Xi} = 0$

Composition of neutron star core

The composition is determined by the rates of transfusion processes ($N = \varnothing, n, p$, $X = n, p, e, \mu$)

$$n + N \to p^+ + \ell + \bar{\nu}_\ell \,, \quad p^+ + N + \ell \to n + \nu_\ell$$

$$e^- + X \to \mu^- + X + \bar{\nu}_\mu + \nu_e \,, \quad \mu^- + X \to e^- + X + \bar{\nu}_e + \nu_\mu$$

- relaxation time $\ll$ time scales of interest
  $\Rightarrow$ equilibrium $\mathcal{A}^\Xi = 0$
- relaxation time $\gg$ time scales of interest
  $\Rightarrow$ frozen composition $\nabla_\mu n_{\rm X}^\mu = 0$

LaTeX

Composition of neutron star core

The composition is determined by the rates of transfusion processes ($N = \varnothing, n, p$, $X = n, p, e, \mu$)

$$n + N \to p^{+} + \ell + \bar{\nu}_{\ell}\,, \qquad p^{+} + N + \ell \to n + \nu_{\ell}$$

$$e^{-} + X \to \mu^{-} + X + \bar{\nu}_{\mu} + \nu_{e}\,, \qquad \mu^{-} + X \to e^{-} + X + \bar{\nu}_{e} + \nu_{\mu}$$

- relaxation time $\ll$ time scales of interest
  $\Rightarrow$ equilibrium $\mathcal{A}^{\Xi} = 0$
- relaxation time $\gg$ time scales of interest
  $\Rightarrow$ frozen composition $\nabla_{\mu} n^{\mu}_{\mathrm{X}} = 0$

In any case

- charge neutrality $n_p = n_e + n_{\mu}$
- conservation of baryon number $\nabla_{\mu} n^{\mu}_{b} = 0$

“Chemical” equilibrium

Chemical affinity of process Ξ

$$\mathcal{A}^{\Xi} \equiv -\sum_{\mathrm{X}} N^{\Xi}_{\mathrm{X}} \mathcal{E}^{\mathrm{X}}$$

N^{Ξ}_{X} particle creation numbers and $\mathcal{E}^{\mathrm{X}}$ energy per particle.

“Chemical” equilibrium

Chemical affinity of process Ξ

$$\mathcal{A}^{\Xi} \equiv -\sum_{\rm x} N^{\Xi}_{\rm x} \mathcal{E}^{\rm x}$$

$N^{\Xi}_{\rm x}$ particle creation numbers and $\mathcal{E}^{\rm x}$ energy per particle.

In multifluid systems, some ambiguity in defining $\mathcal{E}^{\rm x}$. For comoving particles, one can define $\mathcal{E}^{\rm x} = -u^{\mu}\pi^{\rm x}_{\mu}$

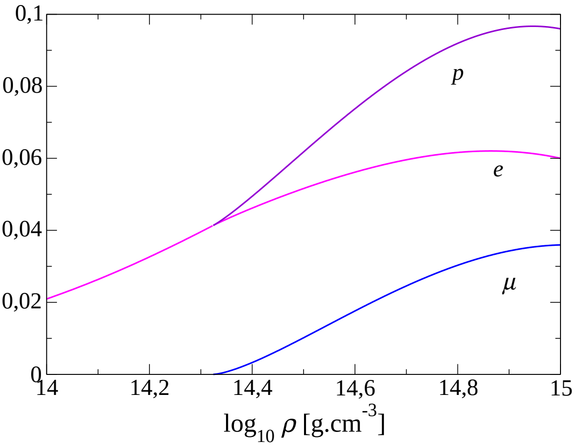
$$\mathcal{A}^{\Xi} = -\sum_{\rm x} N^{\Xi}_{\rm x} \mu_{\rm x} \qquad \mu_{\rm x} = \frac{\partial U_{\rm ins}}{\partial n_{\rm x}}$$

Carter & Chamel,Int.J.Mod.Phys.D14 (2005) 749-774.

LaTeX

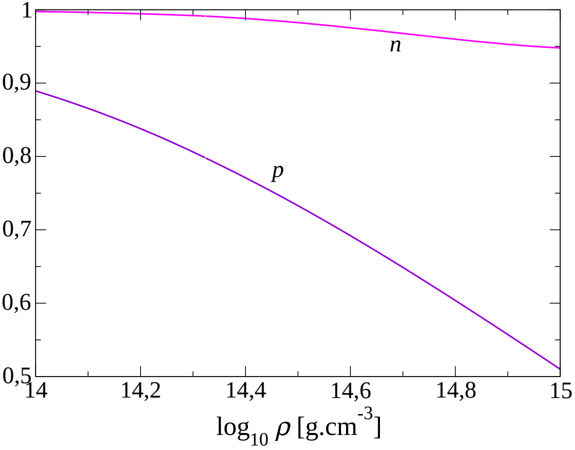
Example : composition of neutron star core

LNS Skyrme force (fitted to Brueckner Hartree-Fock calculations with realistic nucleon-nucleon interactions)



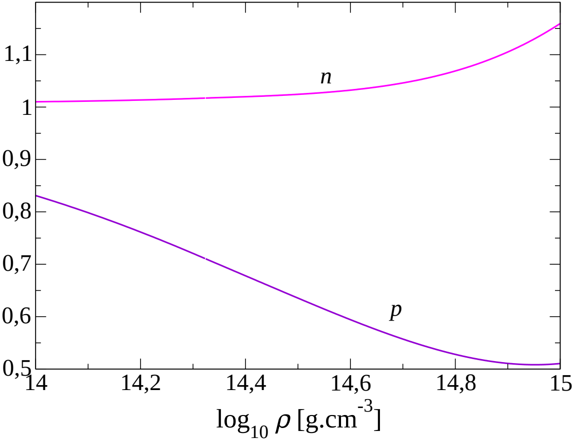
Example : non-relativistic effective masses

LNS Skyrme force (fitted to Brueckner Hartree-Fock calculations with realistic nucleon-nucleon interactions)

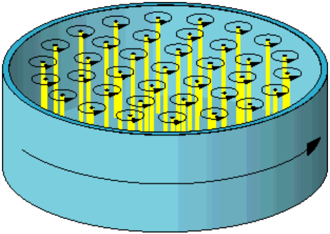


Example : relativistic effective masses

LNS Skyrme force (fitted to Brueckner Hartree-Fock calculations with realistic nucleon-nucleon interactions)



Entrainment and vortices

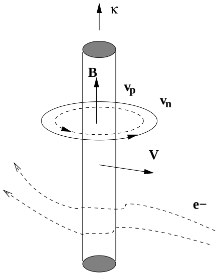


Neutron-proton interactions affect the distribution of neutron vortices

$$n_{\upsilon} = \frac{2m\Omega_n}{h} + \frac{1}{h}(\Omega_n - \Omega_p)\left(\varrho\frac{\mathrm{d}m_\star^n}{\mathrm{d}\varrho} + 2(m_\star^n - m)\right)$$

Chamel & Carter, MNRAS 368 (2006) 796.

Fractional quantum flux of neutron vortices



picture from K. Glampedakis

Due to entrainment, neutron vortices carry a fractional magnetic quantum flux !

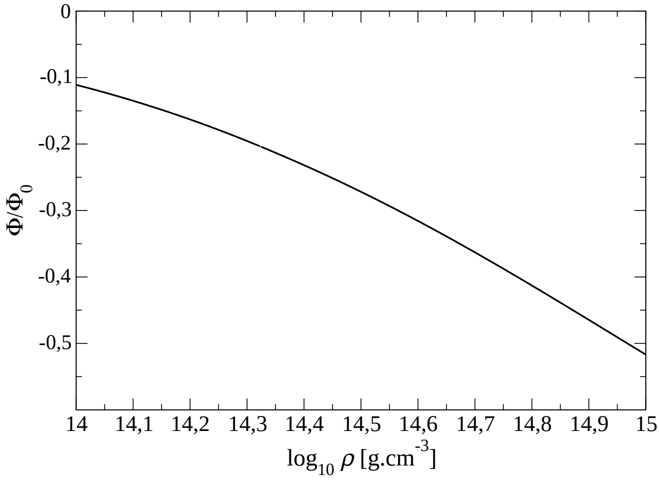
$$\Phi_\star = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} = k\,\Phi_0\,, \qquad \Phi_0 \equiv \frac{hc}{2e}$$

Alpar, Langer, Sauls, ApJ282 (1984) 533-541

LaTeX

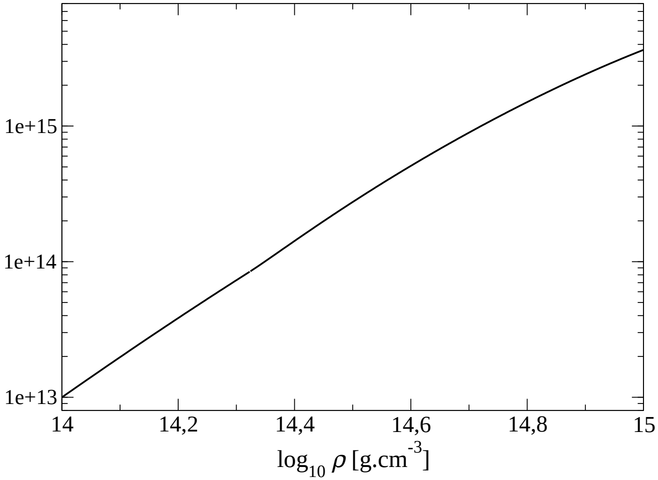
Fractional quantum flux of neutron vortices

LNS Skyrme force



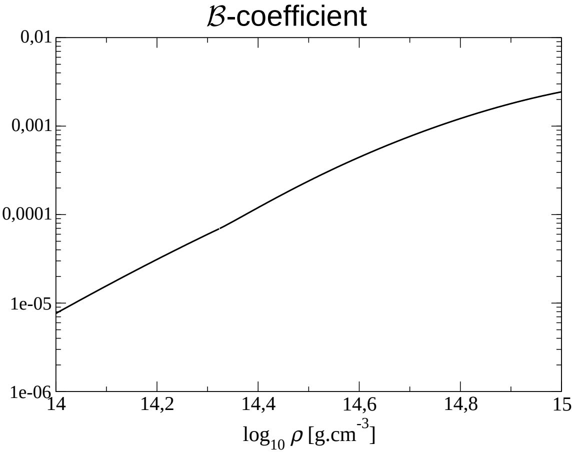
Magnetic field inside neutron vortices

LNS Skyrme force



Mutual friction force

Electron scattering off the magnetic field of the vortex lines leads to a (dissipative) mutual friction force acting on the superfluid.



Conclusion

Superfluidity affects the dynamics of neutron stars
$\Rightarrow$ multi-fluid hydrodynamics

Perspectives :

- construct a unified relativistic elasto-hydrodynamic model of crust and core at finite T (three fluid model)
- calculate consistently *all* microscopic coefficients

Some open issues :

- structure of magnetic field ?
- superconductivity of type I or II ?
- number of fluids in hyperon or quark cores ?
- entrainment effects in such exotic matter ?