Crystallography of Neutron-Star Crusts

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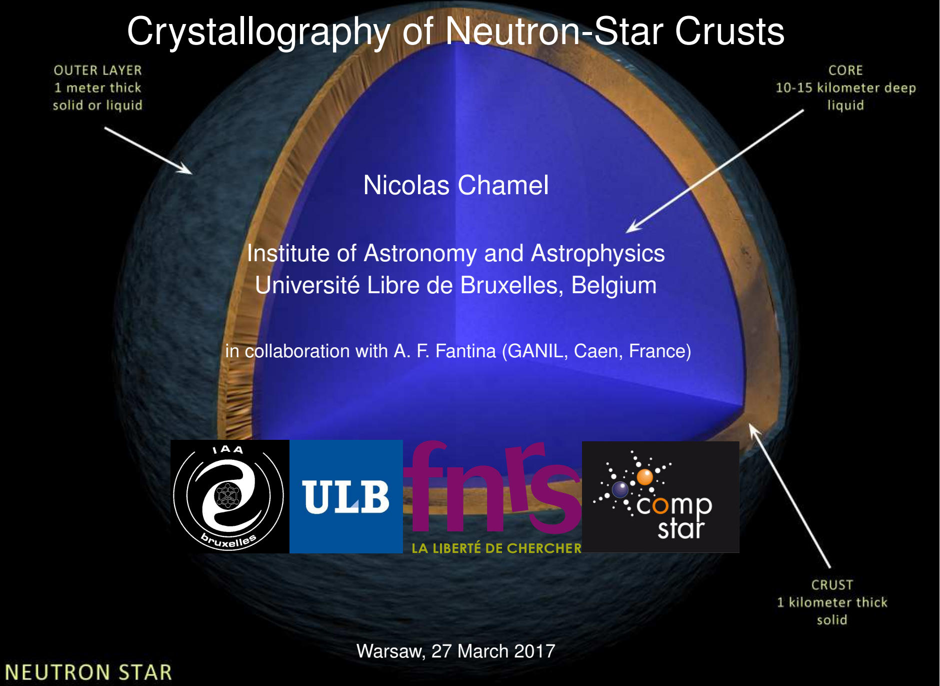
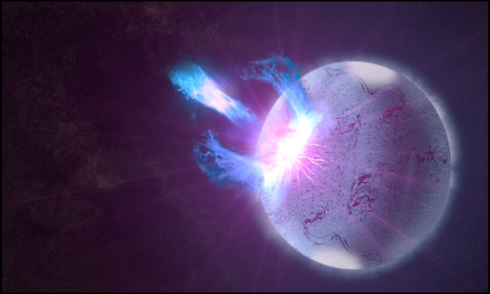


Warsaw, 27 March 2017

Prelude

Many properties of the neutron-star crust are determined by its crystal structure, which in turn have implications for various astrophysical phenomena

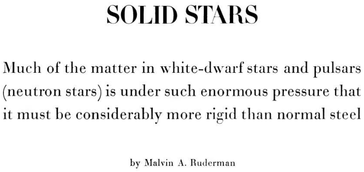
- (small) Crab like pulsar sudden spin-ups,
- thermal relaxation in transiently accreting stars,
- giant flares from soft gamma-ray repeaters and quasiperiodic oscillations,
- gravitational wave emission (mountains)



Credit: NASA's Goddard Space Flight Center/S. Wiessinger

Solid Stars

It has long been assumed that the crust of a neutron star crystallizes into a body-centered cubic lattice, as put forward by M. Ruderman.

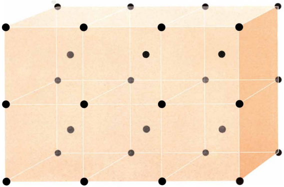


SOLID STARS

Much of the matter in white-dwarf stars and pulsars (neutron stars) is under such enormous pressure that it must be considerably more rigid than normal steel

by Malvin A. Ruderman

Scientific American 224, 24 (1971)



CRYSTAL-LIKE ARRANGEMENT is assumed by nuclei (*black dots*) in superdense solids for the reasons given in the illustration at the top of the page. The arrangement depicted here characterizes matter as it is thought to exist in the central region of white dwarfs and in the outer crust of neutron stars, where the densities range from about a million to 300 trillion grams per cubic centimeter.

His conclusion relied on the **pioneer calculations** of Klaus Fuchs *Proc. Roy. Soc. London A 151, 585 (1935)*.

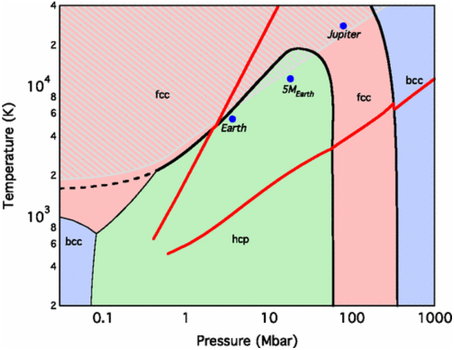
In 1971, Freeman Dyson suggested the existence of **FeHe compound** with rocksalt structure *Ann. Phys. 63, 1 (1971)*.

This possibility was further studied by T. A. Witten *ApJ 188, 615 (1974)*.

Jog & Smith later showed that such a compound is **unstable against weak and strong nuclear processes** *ApJ 253, 839 (1982)*.

Structure of the outermost layers

Compressed iron can be studied with **nuclear explosions and laser-driven shock-wave experiments**... but at pressures corresponding to about 0.1 mm below the surface of a neutron star with a mass $\mathcal{M} = 1.4\mathcal{M}_\odot$ and a radius $R = 12$ km !

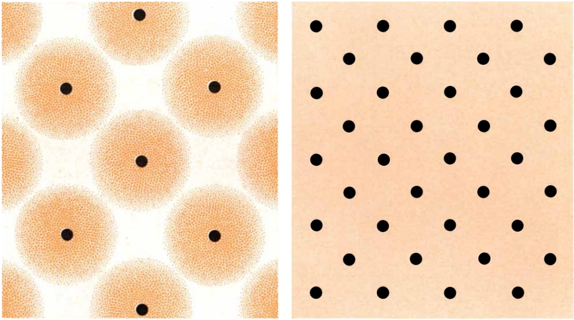


Stixrude, Phys. Rev. Lett. 108, 055505 (2012)

Ab initio calculations predict various **structural phase transitions**.

Crystal Coulomb plasma

At a density $\rho_{\rm eip} \approx 2 \times 10^4$ g cm^{-3} (about 22 cm below the surface), the interatomic spacing becomes comparable with the atomic radius.



Ruderman, Scientific American 224, 24 (1971)

At densities $\rho \gg \rho_{\rm eip}$, atoms are crushed into a **dense plasma of nuclei and free electrons**, expected to form a body-centered cubic lattice crystal.

Description of the outer crust of a neutron star

With further compression, iron becomes unstable against weak and strong nuclear processes.

Main assumptions:

- matter is in full thermodynamic equilibrium
- the crust is stratified into pure layers made of nuclei ${}^{A}_{Z}X$
- electrons are uniformly distributed and are highly degenerate
- nuclei are arranged on a perfect body-centered cubic lattice

$$T < T_m \approx 1.3 \times 10^5 Z^2 \left(\frac{\rho_6}{A}\right)^{1/3} \text{ K} \qquad \rho_6 \equiv \rho/10^6 \text{ g cm}^{-3}$$

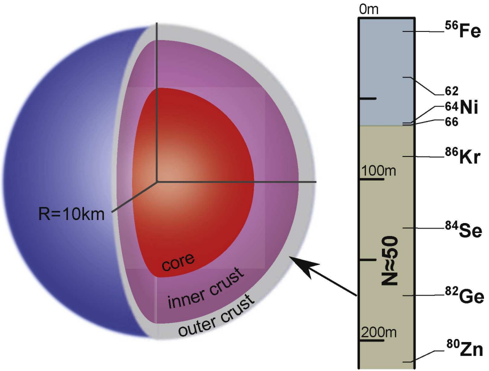
The only microscopic inputs are nuclear masses. We have made use of the experimental data from the Atomic Mass Evaluation complemented with microscopic Hartree-Fock-Bolgoliubov mass tables available at

`http://www.astro.ulb.ac.be/bruslib/`

Pearson,Goriely,Chamel,Phys.Rev.C83,065810(2011)

Composition of the outer crust of a neutron star

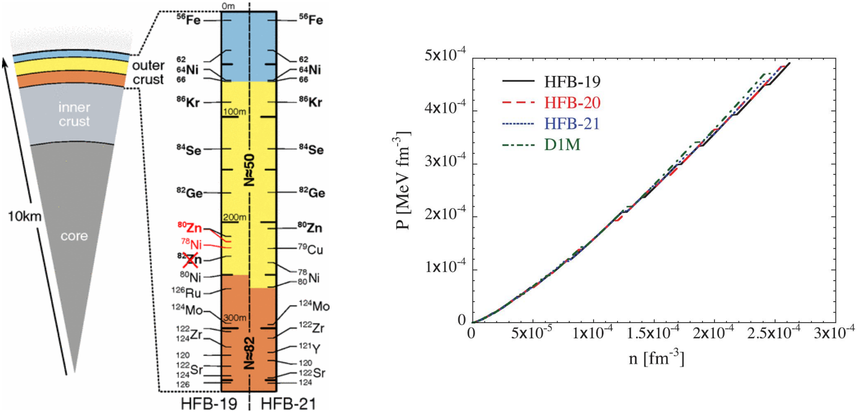
The composition of the crust is completely determined by experimental nuclear masses down to about 200m for a $1.4M_\odot$ neutron star with a 10 km radius



Kreim, Hempel, Lunney, Schaffner-Bielich, Int.J.M.Spec.349-350,63(2013)

Composition of the outer crust of a neutron star

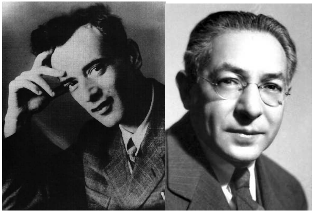
Deeper in the crust, recourse must be made to theoretical nuclear mass models.



Pearson,Goriely,Chamel,Phys.Rev.C83,065810(2011)
Wolf et al.,PRL 110,041101(2013)

Impact of a strong magnetic field on the crust?

In a strong magnetic field $\vec{B}$ (along let's say the z-axis), the **electron motion perpendicular to the field is quantized**:



Landau-Rabi levels

Rabi, Z.Phys.49, 507 (1928).

$$e_\nu = \sqrt{c^2 p_z^2 + m_e^2 c^4 (1 + 2\nu B_\star)}$$

where $\nu = 0, 1, \ldots$ and $\mathbf{B}_\star = \mathbf{B}/\mathbf{B_c}$

with $\mathbf{B_c} = \dfrac{\mathbf{m_e^2 c^3}}{\hbar \mathbf{e}} \simeq \mathbf{4.4} \times \mathbf{10^{13}}$ G.

Maximum number of occupied Landau levels for HFB-21:

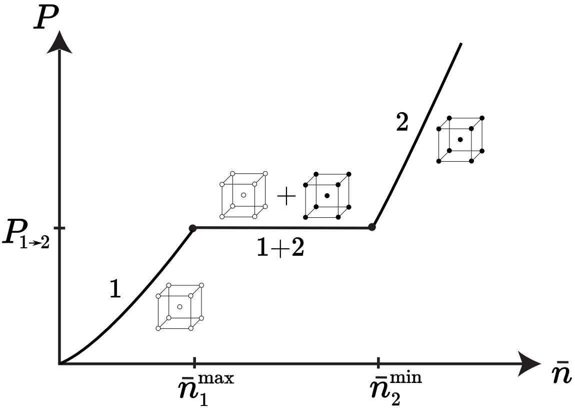
$B_\star$	1500	1000	500	100	50	10	1
$\nu_{\max}$	1	2	3	14	28	137	1365

Only $\nu = 0$ is filled for $\rho < 2.07 \times 10^6 \left(\dfrac{A}{Z}\right) B_\star^{3/2}$ g cm^{-3}.

Landau quantization can change the properties of the crust
(see posters 69&70)

Stratification and equation of state

So far, we have assumed pure layers made of only one kind of nuclei



$$\frac{\bar{n}_2^{\min}-\bar{n}_1^{\max}}{\bar{n}_1^{\max}} \approx \frac{A_2}{Z_2}\frac{Z_1}{A_2}\left[1+\frac{C_{\rm bcc}\alpha}{(3\pi^2)^{1/3}}\left(Z_1^{2/3}-Z_2^{2/3}\right)\right]-1$$

with $C_{\rm bcc} = -1.444231$ and $\alpha = e^2/\hbar c$

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Stratification and equation of state

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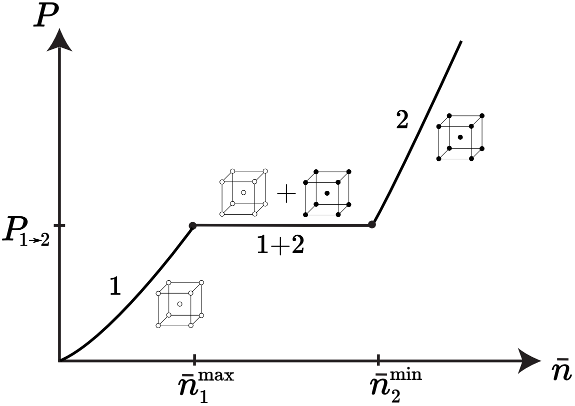
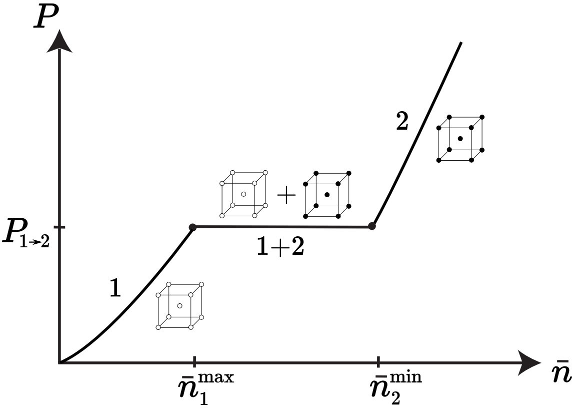


According to Le Chatelier's principle $\bar{n}\dfrac{dP}{d\bar{n}} \geq 0 \Rightarrow \dfrac{\bar{n}_2^{\min} - \bar{n}_1^{\max}}{\bar{n}_1^{\max}} \geq 0$

the denser, the more neutron rich $\dfrac{Z_2}{A_2} \leq \dfrac{Z_1}{A_1}$

Stratification and equation of state

So far, we have assumed pure layers made of only one kind of nuclei



Mixed solid phases cannot exist in a neutron star crust because P has to increase strictly monotonically with $\bar{n}$.

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Compounds in neutron-star crusts?

The structure could be determined using molecular dynamics simulations. However this would be extremely costly because the composition must be also optimized.

Multinary compounds made of nuclei with charges $\{Z_i\}$ could exist in the crust of a neutron star provided

- they are **stable against the separation into pure (bcc) phases**:

  $$\mathcal{R}(\{Z_i/Z_j\}) \equiv \frac{C}{C_{\rm bcc}} f(\{Z_i\}) \frac{\bar{Z}}{\overline{Z^{5/3}}} > 1$$

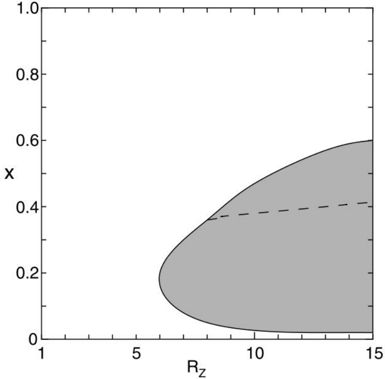
  where $f(\{Z_i\})$ is the dimensionless lattice structure function of the compound and C the corresponding structure constant.
  Chamel & Fantina, Phys. Rev. C 94, 065802 (2016).

This condition is independent of the stellar environment and can thus be easily tested for any given compound structure and composition !

- they are **stable against weak and strong nuclear processes**.

Ordered vs disordered compounds

Stability of disordered binary Coulomb compounds with charge ratios $R_Z = Z_2/Z_1$ and composition $x = N_2/N$

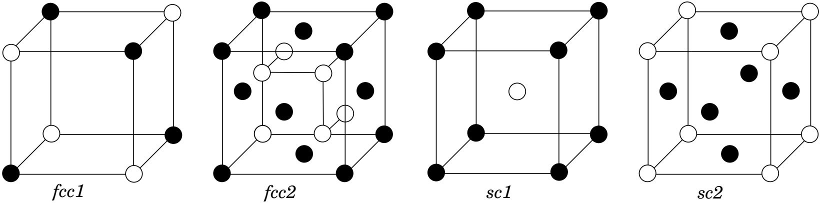


Igarashi, Nakao, and Iyetomi, Contrib. Plasma Phys. 41, 319 (2001).

For the charge ratios $R_Z = Z_2/Z_1 \sim 1$ expected in neutron-star crusts, disordered compounds are unstable.

Binary compounds in neutron-star crusts?

We have investigated the formation of various compounds:



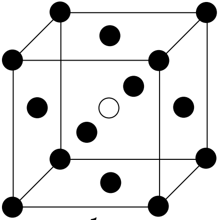
Terrestrial examples:

- fcc1: rocksalt (NaCl), oxydes (MgO), carbonitrides (TiN)
- fcc2: fluorite (CaF_2)
- sc1: cesium chloride (CsCl), β-brass (CuZn)
- sc2: auricupride ($AuCu_3$)

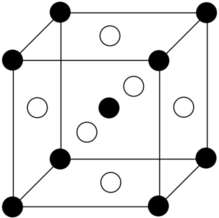
Stellar compounds differ in two fundamental ways from their terrestrial counterparts: (i) they are made of nuclei; (ii) electrons form an essentially uniform relativistic Fermi gas.

Binary compounds in neutron-star crusts?

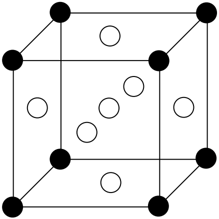
Oher cubic compounds with same structure as perovskites:



p1

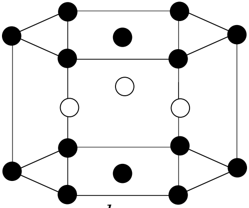


p2



p3

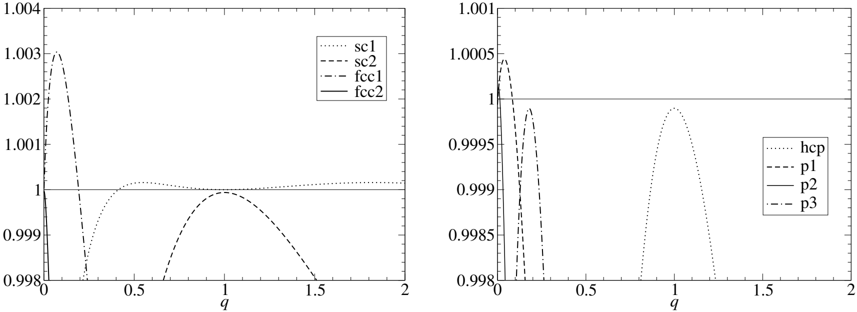
Non-cubic compounds (e.g. tungsten carbide):



hcp

Binary compounds in neutron-star crusts?

Some compounds are unstable against phase separation for any charge ratio $q = Z_2/Z_1$ and can thus be ruled out:

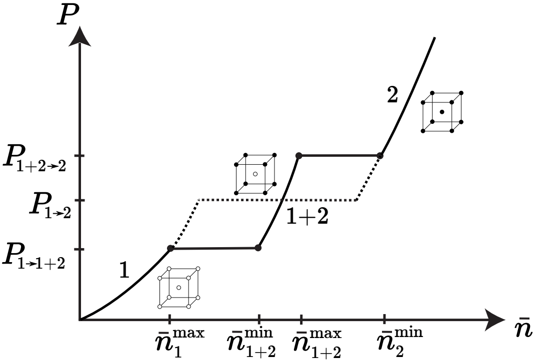


Chamel & Fantina, Phys. Rev. C 94, 065802 (2016).

The most likely compounds are those with CsCl structure.

Substitutional compounds in neutron-star crusts

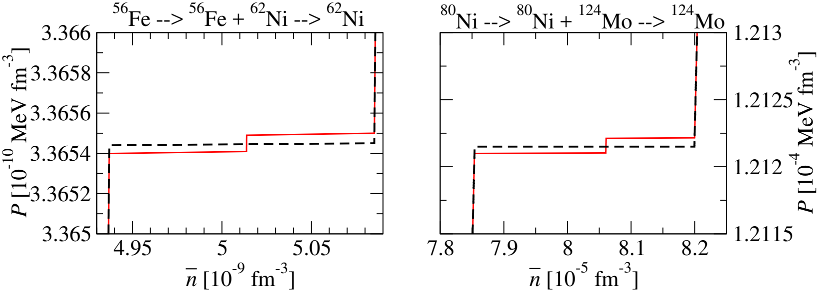
Compounds with CsCl structure are present at interfaces if $Z_1 \neq Z_2$.



$$\frac{\bar n^{\max}_{1+2}-\bar n^{\min}_{1+2}}{\bar n^{\min}_{2}-\bar n^{\max}_{1}} \approx \frac{3C_{\rm bcc}\alpha}{(3\pi^2)^{1/3}}\,\frac{\tilde f(Z_1,Z_2)-\dfrac{\overline{Z^{5/3}}}{\bar Z}}{\left(1-\dfrac{\bar Z A_1}{\bar A Z_1}\right)\left(1-\dfrac{\bar Z A_2}{\bar A Z_2}\right)} \ll 1$$

Substitutional compounds in neutron-star crusts

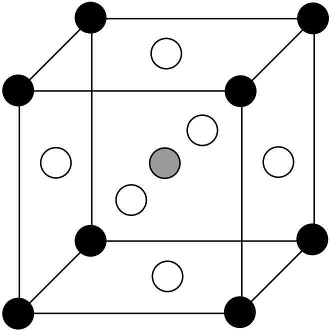
Binary compounds only exist over a very small range of pressures at the interface between two pure adjacent layers.

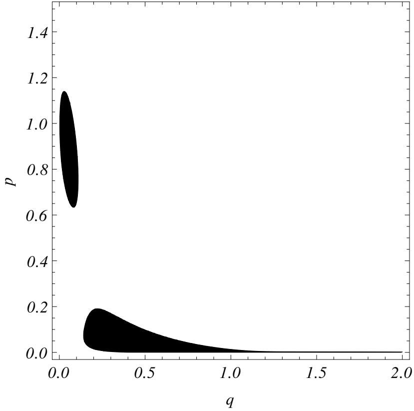


Chamel & Fantina, Phys. Rev. C 94, 065802 (2016).

Ternary compounds in neutron-star crusts?

We have also considered ternary compounds with cubic perovskite structure such as $BaTiO_3$:





No such compounds have been found, but they may exist in accreted crusts.

Conclusions

- it has been generally assumed that the crust of a neutron star is stratified into pure layers with a body-centered cubic lattice structure.
- We have shown that the stability of multinary ionic compounds is uniquely determined by their structure and their composition irrespective of the stellar conditions.
- We have derived very accurate analytical formulas for the transition pressure, as well as for the densities of the different phases irrespective of the degree of relativity of the electron gas
- We have shown that substitutional binary compounds with CsCl structure can form at the interface between two pure adjacent crustal layers.

Multinary ionic compounds may be more abundant in accreted neutron star crusts. Studies are under way.

Chamel & Fantina, Phys. Rev. C 94, 065802 (2016).