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THE EVOLUTIONARY TIMESCALE OF SAKURAI'S OBJECT: A TEST OF CONVECTION THEORY?

Falk Herwig
University of Victoria, Department of Physics and Astronomy, P.O. Box 3055,
Victoria, BC V8W 3P6, Canada; fherwig@uvastro.phys.uvic.ca
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ABSTRACT

Sakurai's Object (V4334 Sagittarii) is a born-again asymptotic giant branch star following a *very late thermal pulse*. So far, no stellar evolution models have been able to explain the extremely fast evolution of this star, which has taken it from the pre–white dwarf stage to its current appearance as a giant within only a few years. A very high stellar mass can be ruled out as the cause of the fast evolution. Instead, the evolution timescale is reproduced in stellar models by making the assumption that the efficiency for element mixing in the He-flash convection zone during the very late thermal pulse is smaller than predicted by the mixing-length theory (MLT). As a result, the main energy generation from fast proton capture occurs closer to the surface, and the expansion to the giant state is accelerated to a few years. Assuming a mass of V4334 Sgr of 0.604 $M_{\odot}$—which is consistent with a distance of 4 kpc—a reduction of the MLT mixing efficiency by a factor of ∼100 is required to match its evolutionary timescale. This value decreases if V4334 Sgr has a smaller mass and, accordingly, a smaller distance. However, the effect does not disappear for the smallest possible masses. These findings may present a semiempirical constraint on the element mixing in convective zones of the stellar interior.

Subject headings: stars: abundances — stars: AGB and post-AGB — stars: evolution — stars: individual (FG Sagittae, Sakurai's Object) — stars: interiors

1. INTRODUCTION

Sakurai's Object (V4334 Sagittarii) has displayed a dramatically fast evolution both in stellar parameters and in chemical abundance patterns (Duerbeck et al. 1997, 2000; Asplund et al. 1999). In 1976, it was possibly detected by the ESO/SERC survey close to the detection limit of $m_J = 21$ (Pollacco 1999), which coincides with the stellar parameters of a pre–white dwarf (WD) in the Hertzsprung-Russell diagram (HRD; see Fig. 1). While this measurement is an important consistency check, the last nondetection in 1994 at the limiting magnitude of $m_V = 15.5$ and the first positive detection at $m_V = 12.4$ by K. Takamizawa in 1995 (see Duerbeck et al. 1997) represent a stringent constraint on the evolutionary speed (Fig. 1). This evolution has been interpreted as the result of a final He flash that occurred in 1994. By early 1996, the star had reached $\log (L/L_{\odot}) \simeq 3.8$ and cooled to well below $\log T_{\rm eff} \sim 4$. Since then, it has continued to cool and brighten while displaying RCrBr-like red declines. Thus, according to the observational evidence, V4334 Sgr must have completed the *born-again evolution* from the pre-WD stage to its current appearance as a giant in about 2 yr. This interpretation is supported by the photoionization modeling of the planetary nebula of V4334 Sgr (Pollacco 1999; Kerber et al. 1999). These models place the central star at an HRD location that is compatible with a pre-WD central star.

The born-again timescale of the related object FG Sagittae has been used successfully to derive the stellar mass under the assumption that the star has gone through a *late thermal pulse* (LTP; Blöcker & Schönberner 1997). Applying the same procedure to V4334 Sgr leads to an extremely large stellar mass of about 1 $M_{\odot}$ as noted by Duerbeck et al. (2000). Such a high mass would require the long distance scale ($d \simeq 8$ kpc) in disagreement with independent distance determinations (such as the extinction method; Kimeswenger & Kerber 1998), which yield distances as low as $d \simeq 1.1$ kpc.

There is more compelling evidence from nucleosynthesis that V4334 Sgr is not very massive. The abundance ratio N/O in the planetary nebula (PN) is well below unity (Pollacco 1999). The PN material reflects the envelope composition during the very last phase on the asymptotic giant branch (AGB). According to recent stellar evolution calculations by Lattanzio & Forestini (1999), AGB stars with the highest mass have a continuously increasing N/O ratio due to hot bottom burning, and eventually the ratio exceeds unity (e.g., at $M_{\rm ZAMS} = 6\ M_{\odot}$ in the case of solar metallicity). Therefore, V4334 Sgr cannot have the highest possible mass.

In fact, V4334 Sgr must be of even lower mass than required by the N/O constraint. Asplund et al. (1999) report a significant lithium abundance that increased over the 6 month period covered by their observation in 1996 (0.5–1.0 dex above initial solar). This amount of lithium cannot be inherited from a previous evolutionary phase but must be a nucleosynthesis product of the special conditions of the final flash (Herwig & Langer 2001). Because any mechanism of lithium creation relies on a readily available reservoir of ^{3}He, the progenitor star must have avoided hot bottom burning altogether, and thus V4334 Sgr is less massive than ∼0.7 $M_{\odot}$.

While, observationally, the very short evolutionary timescale with a probably normal central star of a PN (CSPN) seems well established, a theoretical explanation of this phenomenon is lacking. Here it should be noted that two different kinds of models of the final flash have been constructed. Most models are LTP evolutionary sequences (which apply to FG Sge). The thermal pulse occurs while the star is still on the horizontal crossing from the AGB phase to the CSPN phase at constant luminosity. During this first post-AGB phase, the H shell is still active and prevents the mixing of envelope material into the He-flash convection zone during the thermal pulse. The born-again evolution is energetically driven by the He flash. For these LTP models, the time interval $\tau_{\rm BA}$ from the occurrence of the LTP/VLTP to the return to the AGB is about 100–200 yr (Blöcker 1995; Schönberner 1979). This is clearly in disagreement with the observation of V4334 Sgr.

Both the possible ESO/SERC detection in 1976 and the pho-

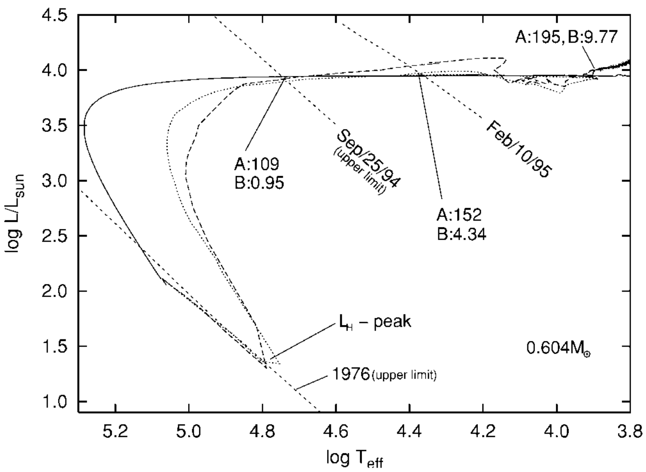

FIG. 1.—HRD of the post-AGB sequence of mass 0.604 $M_\odot$ of Herwig et al. (1999; *solid line*) with the VLTP and subsequent born-again evolution recomputed with $f_v = 3$ (*dotted line*, times labeled "A:") and $f_v = 30$ (*long-dashed line*, times labeled "B:"). The time labels given at $\log T_{\rm eff} = 3.9$ correspond to the first spectra presented in Asplund et al. (1999) that are dated 1996 April 20. All time labels are in years and give the evolutionary time from the moment of the largest H-burning luminosity $L_{\rm H}$ peak. The short-dashed diagonal lines represent three different magnitudes at or below which V4334 Sgr has been observed at different times (see text). In this figure, $d = 4$ kpc has been assumed for internal consistency with the luminosity of the 0.604 $M_\odot$ evolutionary track. The distance has been determined from the luminosity-distance relation derived from the data of Duerbeck et al. (2000) for the date of the first spectra found in Asplund et al. (1999) and the corresponding luminosity of the evolutionary tracks at the temperature determined from the spectra. For the lines of constant magnitudes, the following has been used: for $T_{\rm eff} > 40{,}000$ K BC_V from Napiwotzki (2001) and for $T_{\rm eff} <$ 40,000 K indices from Bessell, Castelli, & Plez (1998; Kurucz atmosphere models), $(V-J) = -0.75$, $A_J = 0.29A_V$, and $E_{B-V} = 0.7$. Two lines are labeled upper limit because V4334 Sgr has been below or close to the detection limit.

toionization models indicate that the post-AGB thermal pulse has occurred very late, when the star has already approached the WD cooling sequence in the HRD. In this case of a *very late thermal pulse* (VLTP), the H burning has stopped, and the protons in the envelope are mixed down into the He-flash convection zone where they burn on the convective timescale. Because the nuclear burning and the convective mixing occur on the same timescale at the same location, a special numerical treatment is required. Iben et al. (1983) circumvented this problem in their model calculation of the VLTP by ignoring the nuclear energy released by proton captures in the He-flash zone. Energetically, their model resembles more that of the LTP, and they found $\tau_{\rm BA}$ to be on the order of a few hundred years (see their Fig. 1), similar to the LTP born-again evolution. Another VLTP model sequence has been presented by Iben & MacDonald (1995) with $\tau_{\rm BA} = 17$ yr. This is closer to the born-again timescale of V4334 Sgr, although still too large by a factor of 3–4. The difference in the two values of $\tau_{\rm BA}$ is probably related to the treatment of hydrogen burning.

The VLTP sequence by Herwig et al. (1999) takes the nuclear energy generation by proton captures in the He-flash convection zone into account. For this purpose, a numerical scheme has been developed that consistently couples the nuclear network equations with the equations of time-dependent convective element mixing (Herwig & Koesterke 2001). However, for this sequence, we found that $\tau_{\rm BA} \sim 350$ yr. Therefore, this model is not in agreement with V4334 Sgr. In this Letter, we demonstrate that the very short born-again evolution time can be reproduced by stellar models if the efficiency of element mixing in the He-flash convection zone is reduced compared with the mixing velocity predicted by the mixing-length theory (MLT).

2. RESULTS

The born-again evolution of V4334 Sgr is another case of the general problem of why stars become red giants. It is well known that the nonlinear stellar structure equations as a boundary value problem have multiple solutions that may be associated with different topologies (e.g., Sugimoto & Fujimoto 2000). If the assumption of thermal equilibrium is relaxed, the transition between solutions of different topologies can be obtained. This leads to the initial value problem of stellar evolution. In order to switch from a dwarf structure to a giant structure, the entropy of the envelope has to be increased.

In the VLTP model of Herwig et al. (1999, Fig. 4), the peak proton-capture energy release is located deep in the He-flash convection zone. The entropy increase in these layers by the additional H-burning luminosity barely affects the outermost layers because, in the He-flash convection zone, the temperature is already greatly increased by the ongoing He flash. If the protons are captured at such a deep position in the intershell region, the corresponding energy release is merely a perturbation of the prominent He-shell instability. Then, the ingestion of protons does not significantly change the timescale of the born-again evolution. As a result, the born-again evolution following a VLTP will be—rather similar to the born-again evolution following the LTP—of the order of a few hundred years. This does not seem compatible with the observed timescale of the born-again evolution of V4334 Sgr.

In order to construct born-again stellar models with an evolutionary speed in agreement with V4334 Sgr, we are looking for a modification of the stellar model that is capable of bringing the position of the main H-energy release from ingested protons closer to the envelope. The position of peak hydrogen burning is determined by the competing mixing ($\tau_{\rm mix}$) and nuclear timescales ($\tau_{\rm nuc}$). In Herwig et al. (1999), we have adopted for the time-dependent treatment of convective element mixing the diffusion coefficient $D_{\rm MLT} = \frac{1}{3}\alpha_{\rm MLT} H_p v_{\rm MLT}$, where $v_{\rm MLT}$ is the convective mixing velocity according to the MLT (Langer, El Eid, & Fricke 1985). The nuclear timescale $\tau_{\rm nuc}$ decreases with increasing temperature as the reaction rate of proton capture by ^{12}C increases. The main energy generation by fast convective proton burning will occur at that position in the He-flash convection zone where $\tau_{\rm nuc} \simeq \tau_{\rm mix}$. This position moves toward the top of the He-flash convection zone if the convective efficiency of mixing the isotopes is reduced. It can be expected that the speed of the born-again evolution after a VLTP depends on the position of H-burning energy release within the star.

In order to evaluate this hypothesis, new VLTP model sequences have been computed using a starting model of the original sequence of Herwig et al. (1999) before the ingestion of protons into the He-flash convection zone begins. The same evolutionary code (EVOL) has been used. The time evolution of 16 isotopes for hydrogen and helium burning is followed. Time-dependent overshoot on any convective boundary can be considered, and the latest OPAL opacities have been used (Iglesias & Rogers 1996). The mixing-length parameter is $\alpha_{\rm MLT} = 1.7$, and the metallicity is $Z = 0.02$.

One way to reduce the convective mixing efficiency would be to change the mixing-length parameter $\alpha_{\rm MLT}$. However, this affects both the convective transport of energy as well as that of matter. According to our hypothesis, we are only interested in the efficiency of material transport. Besides, the mixing-

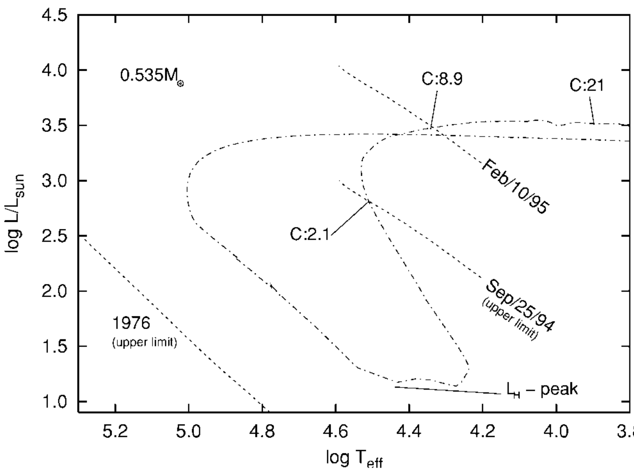

FIG. 2.—Same as Fig. 1, but for a 0.535 $M_{\odot}$ post-AGB evolution with VLTP ($f_v = 1$, times labeled "C:"). Again, the dashed diagonal lines represent different magnitudes at or below which V4334 Sgr has been observed. For this mass, a distance of $d = 2.5$ kpc is consistent with the luminosity of the 0.535 $M_{\odot}$ track.

length parameter describing the efficiency of convective energy transport has been calibrated in order to reproduce the solar parameters. It does not seem plausible to change the well-established parameter $\alpha_{\rm MLT}$. Contrary, the effect of the efficiency of mixing material is much less obvious in stellar evolution calculations. In most situations, the nuclear timescale is larger than the convective mixing timescale by orders of magnitude. For example, for main-sequence stellar models, the velocity of convective material transport can be changed by considerable factors without any change to the stellar parameters. Therefore, we define a new parameter $f_v \equiv D_{\rm MLT}/D_{\rm CM}$, where $D_{\rm CM}$ is the diffusion coefficient for composition mixing. VLTP and born-again model sequences for $f_v = 1$, 3, 30, and 300 have been computed. A 0.535 $M_{\odot}$ VLTP sequence with a $M_{\rm ZAMS} = 1\ M_{\odot}$ progenitor model (Herwig, Blöcker, & Driebe 2000) and $f_v = 1$ has been computed for comparison.

The computations show that the evolution across the HRD is accelerated for larger values of f_v. The cases $f_v = 3$ and $f_v = 30$ for the mass of 0.604 $M_{\odot}$ are shown in Figure 1. The AGB is reached within 195 and 9.77 yr, respectively. In a VLTP model sequence with $f_v = 30$, the peak p-capture energy is released at $m_r \sim 0.601\ M_{\odot}$ compared with $m_r \sim 0.595\ M_{\odot}$ with $f_v = 1$.

Models with reduced convective velocity for composition mixing not only evolve faster but also feature a modified evolution of convective zones. In the new computation with the reduced convective mixing efficiency, H burning takes place in the top layers of the intershell convection zone and establishes its own convective layer on top of the actual He-flash convection zone. In contrast, H burning in the original computation takes place deeper inside the intershell region, and the separate H-burning convection zone inside the He-flash convection zone is very short-lived.

A quantitative comparison of the evolutionary times for different f_v-values with observed evolution times is given in Figure 3 below. Observationally, two time intervals can be defined: from the last nondetection in 1994 to the first positive prediscovery detection in 1995 (t2) and from this time to the date of the first spectra reported by Asplund et al. (1999) (t1). If V4334 Sgr has a mass of 0.604 $M_{\odot}$, then this tentative comparison requires a reduction of the efficiency of composition mixing of $f_v \sim 100$. This follows from the comparison of time interval t1. The fact that time interval t2 requires a larger reduction factor is due to the fact that the observed time interval t2 is only a lower limit because V4334 Sgr was at or below the detection limit in 1994.

3. DISCUSSION

Several tests have been carried out to ensure that obvious sources of uncertainty do not jeopardize the general validity of the findings. Qualitatively, the results are independent of the choice of opacities and the assumptions on overshooting. The influence of numerical parameters, such as the mass at which the outer and the interior solutions are attached, do not change the results qualitatively.

However, several improvements are necessary in future models. We have not considered the μ-barrier during the ingestion of protons, which might effect the results to some extent. Moreover, time-dependent treatment of convective energy transport is not considered. This means that the time step has always to be chosen sufficiently larger than the convective mixing timescale according to the MLT in order to remain consistent with the assumption of instantaneous convective energy transport. This criterion prohibits the usual time resolution that requires, for instance, that the hydrogen-burning luminosity $L_{\rm H}$ may not increase by more than a few percent. In the VLTP case, $L_{\rm H}$ often multiplies by some factor within one time step. However, the nuclear energy integrated by the structure equations and the nuclear energy estimated from the amount of consumed proton agree within 10%–20% in all cases.

The reduced efficiency of composition mixing leads to a different evolution of the convective zones. This affects the CNO element and isotopic ratios. The surface $^{12}C/^{13}C$ ratio of the $f_v = 30$ born-again model is ∼5 and is thus in agreement with the observations of Asplund et al. (1999). Also, the CNO elemental ratios of this sequence are in good agreement with the observed ratios. The absolute CNO abundances of Asplund et al. (1999) are smaller by a factor of ∼5 compared with the model predictions. The reason for this inconsistency is not clear and might be due to the so-called carbon problem of abundance analysis (Asplund et al. 2000).

The formation of lithium is another important test for any stellar model sequence of Sakurai's Object. More detailed studies of the convective nucleosynthesis should provide additional constraints on the validity of the proposed concept. From a preliminary analysis of the temperature conditions in models with reduced convective element mixing, we expect that the mechanism of hot hydrogen-deficient ^{3}He burning (Herwig & Langer 2001) for the synthesis of lithium during the VLTP will provide a lithium abundance in agreement with that of V4334 Sgr.

It has been shown that the born-again evolution following a VLTP is sensitively dependent on the composition mixing due to convection. Therefore, it is important to consider any mechanism or parameter on which the convective velocity in this region depends. The computations show that the convective velocity in the He-flash convection zone decreases with the stellar mass of the post-AGB star. While the 0.604 $M_{\odot}$ sequence displays $v_{\rm MLT} \sim 3$ km s^{-1} (at $\Delta m = 0.003\ M_{\odot}$ below the top boundary of the He-flash convection zone), the comparison model sequence of mass 0.535 $M_{\odot}$ shows only $v_{\rm MLT} \sim 0.35$ km s^{-1}. In accordance with the previous finding, the 0.535 $M_{\odot}$ model sequence does evolve much faster back to the AGB than does the 0.604 $M_{\odot}$ sequence (Fig. 2). This faster evolution must be attributed to the lower convective velocity.

The evolutionary times for the previously described intervals t1 and t2 are included in Figure 3 and lie clearly off the relation between f_v and t_{BA} for 0.604 $M_\odot$. However, note that v_{MLT} of the 0.535 $M_\odot$ sequence is just about a factor of 10 smaller than in the 0.604 $M_\odot$ case. Possibly all these models follow a narrow relation between t_{BA} and v_{CM}.

In any case, the assumption of a small mass for V4334 Sgr probably does not solve the timescale problem alone. The timescale for this model sequence is still too large by an order of magnitude. Moreover, stellar masses as low as 0.535 $M_\odot$ are not consistent with the possible detection in the ESO/SERC survey in 1976 (Fig. 2). However, this argument depends somewhat on the distance-luminosity relation used. Finally, the 0.535 $M_\odot$ evolutionary track is not consistent with the photoionization models of Pollacco (1999) and Kerber et al. (1999). Note that the horizontal luminosity of the 0.604 $M_\odot$ post-AGB track is somewhat larger than that of previous computations of comparable mass because of the inclusion of AGB overshooting and the resulting effects on the core mass–luminosity relation described by Herwig, Schönberner, & Blöcker (1998).

4. CONCLUSION

We have constructed a new born-again stellar evolution model for V4334 Sgr that can reproduce the observed evolutionary speed across the HRD as well as the CNO abundance ratios. It is probably compatible with the observed lithium abundance. For the new model, it has been *assumed* that the efficiency of material transport by convection is smaller than predicted by the MLT by a factor of ∼100 (depending on mass). About the underlying physical origin of such a reduction we can only speculate. The results presented here suggest that a modification to the prescription of convective element mixing may be a solution for the otherwise incomprehensible evolutionary behavior of V4334 Sgr.

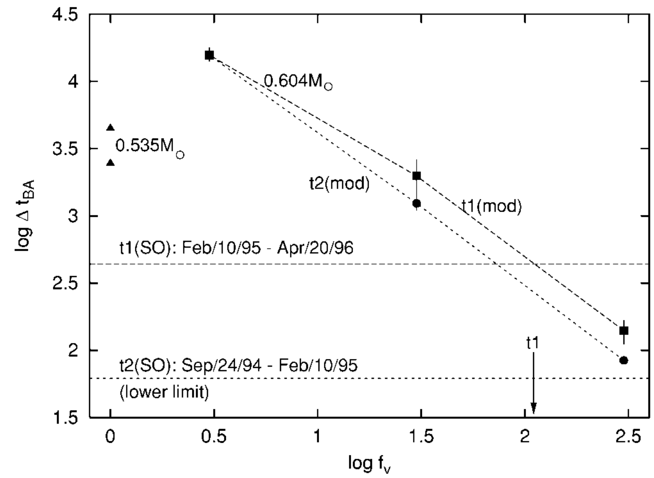

FIG. 3.—Comparison of observed evolutionary speed of V4334 Sgr and of born-again evolution models. The horizontal lines labeled t1(SO) and t2(SO) mark the length of two time intervals between observations of V4334 Sgr. The dates of the observations are given beside the lines (compare Fig. 1). The line t2(SO) is labeled "lower limit" because the corresponding line (Sep/24/94) of constant magnitude in Fig. 1 is an upper limit for V4334 Sgr at that date. From the 0.604 $M_\odot$ model sequences with different values of f_v, the corresponding time intervals have been extracted (*filled symbols*) and connected by lines labeled "t1(mod)" and "t2(mod)." These lines show that born-again models with larger reduction factors evolve faster between two given locations in the HRD. Error bars are plotted for squares only and reflect an estimate of observational uncertainties. The triangles represent the evolutionary times of the 0.535 $M_\odot$ sequence (the top triangle belongs to interval t1). The comparison of the model data and the observed data yields $f_v \sim 100$ if V4334 Sgr has a mass of 0.604 $M_\odot$.

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