

Physics of accretion onto young stars

I. Parametric models

Lionel Siess and Manuel Forestini

Observatoire de Grenoble, Laboratoire d'Astrophysique, Université Joseph Fourier, B.P.53X, F-38041 Grenoble Cedex, France

Received 4 April 1995 / Accepted 4 September 1995

Abstract. The article will present the influence of accreting material on the structure of young stars. We model accretion through a formalism based upon the Richardson criterion which relates the power of buoyancy forces to the kinetic energy of turbulence. In our models we assume that in radiative regions matter is close to marginal stability and we generalize this formalism to convective zones. We take into account the angular momentum as well as the thermal and chemical properties of the accreted matter. We present figures that depict the profile of mass deposition in the star. We study parametrically peculiar aspects of the accretion physics such as how energy is transported through the accretion process or what is the effect of chemical composition differences between the star and the accretion disk or what is the result of accreting more or less angular momentum. The models show that very rapidly the accreted matter can reach the convective zone and eventually can be dragged to the center of the star. This approach will be soon incorporated into evolutionary models of young stellar objects.

Key words: instabilities – pre-main sequence stars – convection – accretion – Richardson number

1. Introduction

Over the past few years, it has become clear that a large fraction of stars undergo accretion during some phases of their evolution but the involved processes are quite different depending on the geometry of the phenomenon. In the case of spherically symmetric infall as can be encountered around protostars, the accreted matter dissipates all its kinetic energy through a strong standing shock at the surface and the energy released heats the central hydrostatic core (Stahler, Shu, & Taam 1980).

Other mechanisms deal with accretion from a disk. Many studies have been concentrated on the late stages of evolution where a white dwarf or a neutron star accrete matter from a

companion filling its Roche lobe. In this scenario, a geometrically thin disk interacts with the star via a boundary layer. In this region, matter is forced to decelerate from an almost Keplerian velocity to the star rotation velocity releasing a large amount of energy of order $L_{acc} \sim GM_*\dot{M}/R_*$, where \dot{M} represents the accretion rate. Although some works (Regev 1984; Regev and Hougerat 1988; Lioure and Leconte 1992) attempt to understand the disk-star boundary, there is still a great uncertainty on what happens exactly. What proportion of the accretion luminosity is imparted to the star or radiated away or what is the maximum depth reached by accreted material?

Previous investigations (Mercer-Smith & al 1984) modeled this phenomenon by considering the boundary layer as a boundary condition in the equation of energy balance, adding to the value of the luminosity at the stellar surface a fraction of the accretion luminosity. Shaviv & Starrfield (1988) first took into account the fact that the accreted matter could bring with it some internal energy but the energy deposition was always assumed to take place at the surface. Palla & Stahler (1992) gauged the effect of accretion geometry by changing the boundary conditions from those of an accretion shock to the ordinary photospheric boundary conditions and qualitatively found only little effects.

However these treatments suppose that matter land “softly” and is simply deposited on the stellar surface. But a more subtil approach of the accretion physics came from the pioneer work of Kippenhahn and Thomas (1978 and hereafter KT) who achieve to giving quantitative results on mass deposition onto a white dwarf. This idea guided Kutter & Sparks (1987 and hereafter KS, 1989) in a more general description of the phenomenon and convinced us that such a mechanism could also hold in the accretion process around young pre-main sequence stars of low mass. Many T Tauri stars are indeed observed to be surrounded by an accretion disk. The formalism developed by KT is based upon the hypothesis that matter is marginally stable with respect to the Richardson criterion. Under this assumption it is possible to determine the profile of the accreted material. This approach allows us to avoid hydrodynamical calculations of the boundary layer and no more consider accretion only through boundary

Send offprint requests to: Lionel Siess

condition changes. Moreover it includes angular momentum, chemical composition and thermal properties of accreted matter.

In the next section, we draw general considerations of instabilities and summarize the general features of the model. In Sect. 3 we present results of this mechanism applied to T Tauri stars and analyze the influence of several parameters. Finally, in Sect. 4. we discuss the incorporation of this formalism in an evolutionary context and conclude.

2. The model

2.1. Shear instabilities

Hydrodynamical instabilities associated with differential rotation have recently received increasing attention. Such instabilities contribute both to angular momentum redistribution and chemical mixing in radiative regions of stars (Knobloch & Spruit 1982; Zahn 1983) and may alter the evolution of the star. In our discussion we will focus our attention on shear instabilities that are likely to be prominent in the accretion process.

In the case of a stable density stratification, the relevant criterion for the onset of such instabilities is the Richardson one. Physically, it is a measure of the competing effects between the stabilizing buoyancy forces and the destabilizing shear forces. Its expression is given by $Ri = N^2 / (dU/dz)^2$ where N is the Brunt-Väisälä frequency, U the horizontal flow velocity and z the direction perpendicular to the flow. To take into account compressibility, Sung (1974) derived in the case of differentially rotating homogeneous cylindrical stars a more general criterion that includes the Schwarzschild condition

$$Ri = \frac{\tilde{g}}{H_p} \frac{\nabla' - \nabla}{(\varpi \frac{d\Omega}{d\varpi})^2}, \quad (1)$$

where \tilde{g} is the effective gravity, ϖ the radial distance from the rotation axis, Ω the angular velocity and H_p the pressure scale height. ∇' and ∇ are the average temperature gradients inside the element and in the outside local matter, respectively. In case of adiabatic motion, we have $\nabla' = \nabla_{ad}$. In our context, ∇' refers to the accreted material and will be noted ∇^a .

The criterion states that if $Ri > 1/4$ everywhere, the accreting flow is stable (Chandrasekhar 1961). However this condition holds in the limit of homogeneous, inviscid and more generally non-dissipative flows. In fact, observations of Earth's atmosphere indicate that instabilities are occurring in regions for which the Richardson criterion predicts strong stability. In order to explain this, we have to consider thermal diffusion and viscosity. Their effects are to smooth out the temperature fluctuations and hence density perturbations. As a result, the stabilizing buoyancy forces are weakened and in some case can no longer prevent the muted instability. To account for this, we will follow the idea developed by Townsend (1958). If the cooling time scale t_{cool} associated with the turbulent element is much smaller than the turn-over time scale $t_{to} \sim (dU/d\varpi)^{-1}$, the critical Richardson number is multiplied by a factor t_{to}/t_{cool}

(Schatzman & Praderie 1990). If the accreted globule has a typical size l then $t_{cool} \sim l^2/K$ where K is the thermal diffusivity. The conjecture proposed by Zahn (1974) is to identify l to the smallest scale that is shearily unstable, so that

$$l^2/\nu dU/dz = Re_c,$$

where Re_c is the critical Reynolds number and ν the cinematic viscosity. Using the Prandtl number $Pr = \nu/K$, we obtain

$$t_{cool}/t_{to} = Pr Re_c = Pe,$$

where Pe is the Peclet number associated with the globule that will be noted Pe^a . In this context Pe^a must be below unity and the thermal part of the buoyancy force now writes

$$(\nabla^a - \nabla) = Pe^a (\nabla_{ad} - \nabla). \quad (2)$$

Concurrently, treatment of non-adiabatic convection tells us that (Cox & Guili, 1968)

$$\nabla^a - \nabla = \frac{\Gamma^a}{\Gamma^a + 1} (\nabla_{ad} - \nabla) = \mathcal{E}^a (\nabla_{ad} - \nabla), \quad (3)$$

where Γ^a is the efficiency of the accretion energy transport. Identification of relations (2) and (3) tells us that in the approximation of important radiative losses ($Pe^a \ll 1$), $Pe^a = \frac{\Gamma^a}{\Gamma^a + 1}$. However, in the general case it can be shown (Maeder 1995) that the thermal part of the buoyancy force becomes

$$\nabla^a - \nabla = \frac{Pe^a}{Pe^a + \frac{Al}{V}} (\nabla_{ad} - \nabla), \quad (4)$$

where A and V are the surface and volume of the element. In the literature Al/V is usually taken equal to 6. In the limiting case of large value of Pe^a , $\nabla^a \rightarrow \nabla_{ad}$ and we recover expression (1) in case of adiabatic accretion. On the opposite, if radiative losses are important $Pe^a \sim \mathcal{E}^a \rightarrow 0$ and $\nabla^a \rightarrow \nabla$ indicating that the accreted matter is thermalized.

In addition, if a chemical composition gradient is present, it must be included into the criterion. Such a μ -gradient has generally a stabilizing influence because it sustains a density gradient. Finally, Ri reads

$$Ri = \frac{\tilde{g}}{H_p} \frac{(\nabla_{ad} - \nabla) \mathcal{E}^a + \nabla \mu}{(\varpi \frac{d\Omega}{d\varpi})^2}, \quad (5)$$

where $\nabla \mu$ is the mean molecular weight gradient that only refers to differences in chemical composition, excluding thermodynamic effects such as ionization that are already taken into account in the evaluation of the adiabatic gradient. We can notice that the numerator is to compare to Ledoux criterion for dynamical stability, usually involved to treat stellar convection.

Through all this section we assume the Brunt-Väisälä frequency to be positive, i.e. to stand in a radiative region with a stable density stratification. However young stars are almost entirely convective, presenting only a very thin radiative envelope. In this convective zone, the Schwarzschild criterion is violated ($\nabla_{ad} < \nabla$) and N^2 (the numerator) becomes negative.

In such a situation, we will impose the Richardson number to be negative and name it the *Richardson number for convection* Ri^- . Small negative values of this parameter correspond to a turbulent regime where the convection is not very efficient but as this number increases (in absolute value) one draws near to free convection (Favre et al 1976).

2.2. Mathematical formulation

2.2.1. The Richardson number

As mentioned before, the generalized Richardson number represents the proportion of energy imparted to buoyancy and centrifugal forces with respect to turbulence and its expression is given by

$$Ri = \frac{\text{Potential energy of buoyant and centrifugal forces}}{\text{Kinetic energy available from differential rotation}}$$

To evaluate this number in the case of accretion, we assume the time scale for turbulent mixing is short compare with the accretion time scale ($\tau_{acc} \sim M_{acc}/\dot{M}$) where M_{acc} is the accreted mass, i.e. the surface layers (radiative layers) are always close to marginal stability or more formally $Ri \sim 1/4$. We further assume that (i) accretion is a small distortion of the star, (ii) equipotential surface are mainly spherical and (iii) the problem can be treated as axially symmetric.

The turbulent element is in pressure equilibrium with its surrounding and for a displacement $\Delta \mathbf{l}$ in the meridional plane, the resulting force is, to first approximation, parallel to $\Delta \mathbf{l}$

$$\mathbf{F} = \left[\frac{D\rho}{\rho} (\mathbf{g} \cdot \mathbf{e}_{\Delta \mathbf{l}}) + \frac{D(r\omega^2\rho)}{\rho} (\mathbf{e}_{\omega} \cdot \mathbf{e}_{\Delta \mathbf{l}}) \sin \theta \right] \Delta \mathbf{l}, \quad (6)$$

where D evaluates the difference of a quantity \mathcal{A} between the accreted element (subscript e) and the surroundings ($D\mathcal{A} = \mathcal{A}_e - \mathcal{A}$), \mathbf{e}_{ω} is the unit rotating vector perpendicular to the rotation axis and $\mathbf{e}_{\Delta \mathbf{l}}$ the unit vector along $\Delta \mathbf{l}$. The work performed over the distance l is $W = \int_r^{r+l} \mathbf{F} \cdot d(\Delta \mathbf{l})$. After expanding expression (6) to first order we obtain

$$W = -\frac{1}{2} l^2 \left[\left(\frac{GM_r \cos \alpha}{r^2} - \omega^2 r \sin(\theta + \alpha) \sin \theta \right) \frac{\rho'_e - \rho'}{\rho} - 2\omega^2 r \sin(\theta + \alpha) \sin \theta \frac{\omega'_e - \omega'}{\omega} \right], \quad (7)$$

where α is the angle between the radial direction \mathbf{r} and $\Delta \mathbf{l}$. In a stable region, this quantity is negative and reflect the fact that the buoyant force is a restoring force.

The kinetic energy resulting from differential rotation is

$$E = \frac{1}{2} l^2 [(\omega'_e - \omega') r \sin \theta]^2. \quad (8)$$

In Eqs. (7) and (8), primes refer to derivative along the motion¹. By assuming angular momentum conservation,

$$\frac{\omega'_e}{\omega} = -\frac{2 \sin(\theta + \alpha)}{r \sin \theta}. \quad (9)$$

Following the conjecture proposed by Zahn (1974) and reformulated by Maeder (1995) we can write

$$\frac{\rho'_e - \rho'}{\rho} = \left(\frac{Q}{H_p} (\nabla_{ad} - \nabla) \mathcal{E}^a - \frac{\phi}{H_p} \nabla \mu \right) \cos \alpha + \frac{\phi}{r} \frac{d \ln \mu}{d \theta} \sin \alpha, \quad (10)$$

where Q and ϕ are thermodynamics quantities, namely

$$Q = -\left(\frac{d \ln \rho}{d \ln T} \right)_P \quad \text{and} \quad \phi = -\left(\frac{d \ln \rho}{d \ln \mu} \right)_{P,T},$$

where μ is the mean molecular weight per ion. This expression is derived by assuming equipotential surfaces to be roughly spherical, i.e. ρ -variations in latitude are solely the fact of gradients in chemical composition.

The Richardson number is then given by

$$Ri = \frac{\left(\frac{GM_r \cos \alpha}{r^2} - \omega^2 r \sin(\theta + \alpha) \sin \theta \right) \frac{\rho'_e - \rho'}{\rho}}{((\omega'_e - \omega') r \sin \theta)^2} - \frac{2\omega^2 r \sin(\theta + \alpha) \sin \theta \left(\frac{\omega'_e - \omega'}{\omega} \right)}{((\omega'_e - \omega') r \sin \theta)^2}. \quad (11)$$

2.2.2. Definitions

Following KS, we define a function f which both represents the fraction of mass and angular momentum deposited at each radius r . The idea is that angular momentum is transported into the star by the accreted material. This definition is translated by

$$j(r, \theta) = r^2 \omega(r, \theta) \sin^2 \theta \equiv \xi j_K f(r, \theta), \quad (12)$$

where $j_K = (GM_* R_*)^{1/2}$ is the Keplerian specific angular momentum at the surface of the star, ξj_K its value at the equatorial surface ($0 < \xi \leq 1$) and f a penetration function ($0 < f \leq 1$) that we define as

$$f \equiv \frac{dm_r^a}{dm_r^*}, \quad (13)$$

where dm_r^a and dm_r^* represent the amount of accreted material deposited between radii r and $r + dr$ and the mass of this shell before accretion, respectively. This function satisfies

$$\int_0^{R_*} f dm_r^* = M_{acc} = \dot{M} \Delta t. \quad (14)$$

This normalization implies that all the accreted angular momentum is redistributed in each mass shell to stellar matter.

¹ For any function $f(r, \theta)$,

$$f' = \left(\frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \mathbf{e}_\theta \right) \cdot \mathbf{e}_{\Delta \mathbf{l}} = \cos \alpha \frac{\partial f}{\partial r} + \sin \alpha \frac{1}{r} \frac{\partial f}{\partial \theta}.$$

2.2.3. Determination of $\nabla\mu$ and ω'

The mean molecular weight of a gas inside a mass shell m is

$$\begin{aligned}\mu^{-1} &= \mu_{ion}^{-1} + \mu_e^{-1} = \sum_i X_i \frac{m_H}{m_i} + \frac{N_e m_H}{m} \\ &= \frac{m_H}{m} \left(\sum_i N_i + N_e \right),\end{aligned}$$

where N_i is the number of species i , N_e the number of electrons, m_i the mass of atom i , X_i the mass fraction of species i and m_H the mass of hydrogen. As we pointed out before, $\nabla\mu$ in Eq. (10) only deals with variation of chemical composition so we only retain the ionic component. Let us now establish the μ_{ion} change due to accretion. During mixing, the number of particle i at a depth r ($N_{i,r}^*$) is increased by the arriving accreted matter. After accretion $N_{i,r}^{*+a} = N_{i,r}^* + N_i^a$ and $dm_r^{*+a} = dm_r^* + dm_r^a$. The new ionic mean molecular weight is then

$$\begin{aligned}\frac{1}{\mu_{ion,r}^{*+a}} &= \frac{m_H}{dm_r^{*+a}} \sum_i N_{i,r}^{*+a} = \frac{dm_r^*}{dm_r^{*+a}} \frac{1}{\mu_{ion,r}^*} + \frac{dm_r^a}{dm_r^{*+a}} \frac{1}{\mu_{ion,r}^a} \\ &= \frac{1}{1+f} \left(\frac{f}{\mu_{ion,r}^a} + \frac{1}{\mu_{ion,r}^*} \right),\end{aligned}\quad (15)$$

where $\mu_{ion,r}^a$ and $\mu_{ion,r}^*$ are the ionic mean molecular weight of the accreted material and of the star at depth r , respectively. To derive this expression we have assumed $\mu_{ion,r}^a$ constant through the displacement, i.e. the turbulent element keeps its identity until it dissolves.

After some algebra, we obtain

$$\nabla\mu^{*+a} = H_p \frac{\left(\frac{1}{\mu_{ion}^a} - \frac{1}{\mu_{ion,r}^*} \right)}{\frac{(1+f)^2}{\mu^{*+a}}} f' + \frac{\nabla\mu}{1+f} \frac{\mu^*}{\mu^{*+a}}, \quad (16)$$

where μ^* and μ^{*+a} are the mean molecular weight before and after the accretion process, respectively.

The expressions of ω' are derived from Eq. (12)

$$\frac{\partial \ln \omega}{\partial \ln r} = \frac{\partial \ln f}{\partial \ln r} - 2 \quad (17)$$

$$\frac{\partial \ln \omega}{\partial \theta} = \frac{\partial \ln f}{\partial \theta} - 2 \cot \theta. \quad (18)$$

2.2.4. The penetration function in the horizontal direction

In this section we write the equation for f only taking into account its horizontal component ($\alpha = \pi/2$) that we will note $f_t = f(\theta, \alpha = \pi/2)|_r$. From Eqs. (9), (10), (11), (18) we obtain at each radius r for the tangential component

$$\frac{\partial \ln f_t}{\partial \theta} = \frac{2}{Ri} \cot \theta \left[1 + \frac{\phi \left(\frac{1}{\mu_{ion}^a} - \frac{1}{\mu_{ion}^*} \right)}{\frac{(1+f_t)^2}{\mu^{*+a}}} \right]. \quad (19)$$

In our case the chemical composition of the accreted matter do not differ significantly from that of the star so that in particular, Eq. (19) can reduce to

$$\frac{\partial \ln f_t}{\partial \theta} = \frac{2}{Ri} \cot \theta. \quad (20)$$

This gives after integration, for $Ri = 1/4$

$$f_t(r, \theta) = f_t(r, \pi/2) \sin^8 \theta,$$

so that there is no mass deposition at the pole of the star. This expression only gives an estimate of the behavior of f_t in the horizontal direction and cannot be considered as formal considering the arbitrariness of Ri in the surface layers and depending on the transport process.

Following KT, we simulate the geometry of accretion by defining θ_H the angle at which

$$f_t(r, \theta_H) = \epsilon f_t(r, \pi/2) = f_t(r, \pi/2) \sin^8 \theta_H \quad (21)$$

where $0 \leq \epsilon < 1$. We define also $\phi_H = \pi/2 - \theta_H$, the width of the accretion belt above the equator. Note that a spherical accretion is represented by $\epsilon = 0$ or $\phi_H = \pi/2$. This angle depends on the horizontal mixing time scale. In the white dwarfs envelope, MacDonald (1983) found that in the case of non-axisymmetric perturbations, mixing in the horizontal direction occurs in a dynamical time scale. Then if the accretion time scale is longer than the dynamical one, the accreted material will spread over the whole surface of the star. In a convective region, horizontal mixing is assured by the rapid turbulent motion of the fluid. Thus ϕ_H should be close to $\pi/2$. As we will see however, its influence is very weak.

2.2.5. The penetration function in the radial direction

We will now write the equation for f only taking into account its radial component that we will note as $f_\omega = f(r, \alpha = 0)|_\theta$. Using the same equations as before we obtain at each radius r

$$R_* Ri f_\omega'^2 + B(f_\omega) f_\omega' + C(f_\omega) = 0, \quad (22)$$

with

$$B(f_\omega) = \left(\frac{M_r \sin^2 \theta}{\xi^2 M_*} - \frac{R_*}{r} f_\omega^2 \right) \frac{\phi \left(\frac{1}{\mu_{ion}^a} - \frac{1}{\mu_{ion}^*} \right)}{\frac{(1+f_\omega)^2}{\mu^{*+a}}} - 2 \frac{R_*}{r} f_\omega,$$

and

$$C(f_\omega) = \left(\frac{M_r \sin^2 \theta}{\xi^2 M_*} - \frac{R_*}{r} f_\omega^2 \right) \left[\frac{\phi \nabla\mu}{H_p \mu^*} - \frac{Q}{H_p} (\nabla_{ad} - \nabla) \mathcal{E}^a \right].$$

This yields to

$$\frac{\partial f_\omega}{\partial r} = - \frac{B(f_\omega)}{2 Ri R_*} + \left| \frac{\sqrt{B(f_\omega)^2 - 4 R_* Ri C(f_\omega)}}{2 Ri R_*} \right|. \quad (23)$$

We always select the positive value for f'_{ϖ} because it does not appear physical that matter can accumulate in some regions inside the star. Moreover, taking $f'_{\varpi} > 0$ avoid the Rayleigh-Solberg instability: $d(\varpi^2\omega)^2/d\varpi > 0$ (MacGregor & Brenner 1991).

However, we have to keep in mind that all variables entering Eq. (23) will come from a one-dimensional code and dependence in θ cannot be taken into account. We thus have to average all quantities over angle. If we assume that the equatorial belt rotates uniformly as a function of radius only, (i) the penetration function must be replaced by

$$\begin{aligned} \langle f_{\varpi} \rangle_{\theta} &= \omega(r) r^2 \langle \sin^2 \theta \rangle \\ &= \omega(r) r^2 \frac{\int_{\theta_H}^{\pi/2} \sin^2 \theta \sin \theta d\theta}{\int_{\theta_H}^{\pi/2} \sin \theta d\theta} \\ &= f_r \Theta \quad \text{with } f_r = \omega(r) r^2, \end{aligned} \quad (24)$$

and (ii) in Eq. (22), the centrifugal force must be averaged over the width of the accretion belt, i.e. $\sin^2 \theta$ replaced by Θ . Moreover, (iii) all integrations connected with accretion must be multiplied by the ratio of the solid angle subtended by the belt to 4π .

3. Numerical results

3.1. Description of the selected stellar model

We start our stellar evolution calculations with initial models that lies far above the birth line described by Stahler (1988). In the forthcoming parametric study, all variables entering Eq. (23) are taken from such an initial model, the mass of which is $0.2M_{\odot}$. The metallicity is taken to be solar. This model is located in the Hertzsprung-Russell diagram at the upper most branch of the convective Hayashi track with a luminosity $L \sim 12L_{\odot}$ and a radius of $10.7R_{\odot}$. It is entirely convective and is surrounded by a thin radiative envelope of $\sim 10^{-6}M_{\odot}$. The photospheric and central temperatures are 2900 K and 2×10^5 K, respectively.

3.2. Role of parameters

The formalism here developed provides us a large area of exploration of the physics of accretion that still remains largely unknown. With our set of parameters, it is possible to analyze the effects of heat transport of the accreted matter through Pe^a , to gauge the efficiency of the accretion power by selecting different Richardson number of convection Ri^- or to study the influence of accreting more or less angular momentum by changing ξ .

3.2.1. The influence of chemical composition

Chemical composition of young stars has no reason to differ significantly from that coming from accretion disk because both originate from the same cloud, so μ^a will not play an important role. For information we present in Fig. 1 the influence of this parameter in the profile of f . As expected, the denser the accreted matter (i.e. the larger is μ^a) the deeper matter sinks into the star.

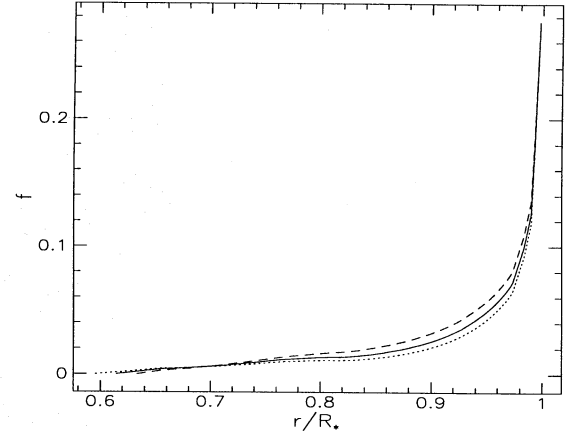


Fig. 1. Effect of varying chemical composition on the penetration function. The solid line refers to accreted chemical composition equal to that existing at the surface of the star (μ_s^*), while the dotted and short-dashed lines correspond to μ^a values that differ by more or less 20% compared to μ_s^* , respectively. Parameters are set to $\mathcal{E}^a = 0.5$, $\epsilon = 0.5$, $\xi = 0.5$, $Ri^- = -10$ and $M_{acc} = 10^{29}$ g.

However, when nucleosynthesis will start, it is interesting to follow the abundance of peculiar elements such as ^2H , ^3He , ^7Li or ^9Be . In particular, ^7Li reveals to be of great importance in evaluating stellar ages. Although depletion of ^2H and ^7Li do not affect the mean molecular weight, its effect on observed abundances can be relevant. The disk indeed acts as a tank of fresh material with constant chemical composition and during all the accretion phase various elements, though destroyed in the star, will be accreted.

3.2.2. The influence of the accretion rate

Given a time step Δt , the influence of the accretion rate reflect the amount of mass deposited in the star. We see in Fig. 2 that the accreted material progressively penetrates into the star and very rapidly reaches the convective region.

The different curves reflect how accreted matter would distribute if stellar structure was not perturbed by the accretion process. We recall that angular momentum is deposited proportionally to mass.

3.2.3. The role of Pe^a

Before exploring the effect of this parameter, let us come back to its signification. This number represents the ratio of a cooling time scale to the turn-over time scale. Its value ranges between zero when the accreted element is thermalized very quickly ($t_{cool} \ll (dU/dz)^{-1}$) and infinity for an adiabatic accretion. As noticed by KS, accretion process acts as a refrigerator cycle by transporting energy inward against the natural temperature gradient of the ambient gas. In the limit of thermal equilibrium

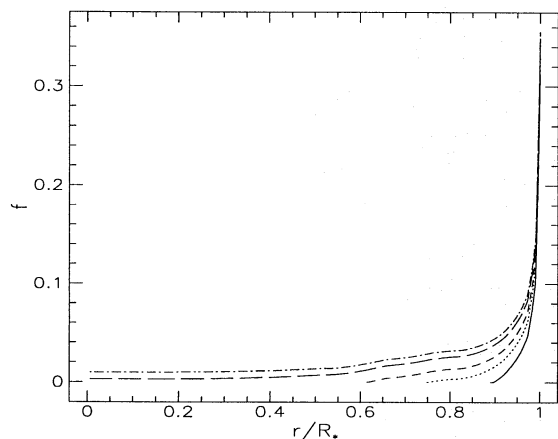


Fig. 2. f as a function of M_{acc} . The solid line corresponds to $M_{acc} = 10^{27}$ g, the dotted, short-, long- and dotted short-dashed lines to $M_{acc} = 10^{28}$ g, 10^{29} g, 10^{30} g and 2×10^{30} g, respectively. Parameters are the same as before with $\mu^a = \mu_s^* - 0.01$.

between the element and its surroundings the work is minimized and mixing is only the response of angular momentum transport and chemical composition differences. With Pe^a it is possible to describe the way energy is transported during accretion and its influence on the penetration function is shown in Fig. 3. We notice that when matter is rapidly thermalized (Pe^a close to zero), it penetrates very deep into the star as it presents less opposition to the inwards convective motions. The accreted material is freely transported by convection. Conversely, if accretion proceeds adiabatically, Pe^a is large and \mathcal{E}^a is close to unity. This means that heat transport by the accretion flow is very efficient and able to interfere with the convective motions. In such a case, the accreted material can penetrate less inside the star.

3.2.4. The Richardson number in the convective zone

As was pointed out in the previous section, the Richardson number in the convective zone must take a negative value. In the phenomenologic treatment of convection (Böhm-Vitense, 1958), a convective element forms and after moving a length Λ dissolves and releases its heat excess. It is assumed that all elements have the same average velocity which is derived by equating the work done against the buoyancy forces with the kinetic energy of the globule. The convection power is then comparable to the accretion one except for the inclusion of angular momentum in the latter case. It seems reasonable to think that Ri^- increases as the efficiency of convection Γ does. This latter is defined by the following expression (Cox and Guili, 1968)

$$\Gamma \equiv \frac{\text{"Excess heat content" just before dissolving}}{\text{Energy radiated during lifetime}}.$$

Ri^- expresses the destabilizing capability of the convection power. The energy radiated away during life time is a source

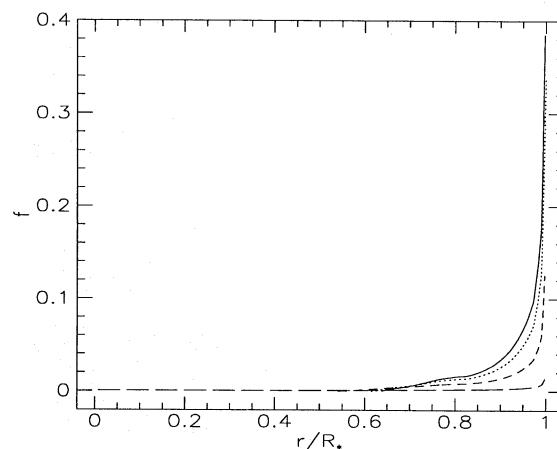


Fig. 3. Profile of f for different values of \mathcal{E}^a : 1 (solid line), 0.5 (dotted line), 0.1 (short-dashed line) and 0.001 (long-dashed line). The parameters are the same as before with $M_{acc} = 10^{29}$ g.

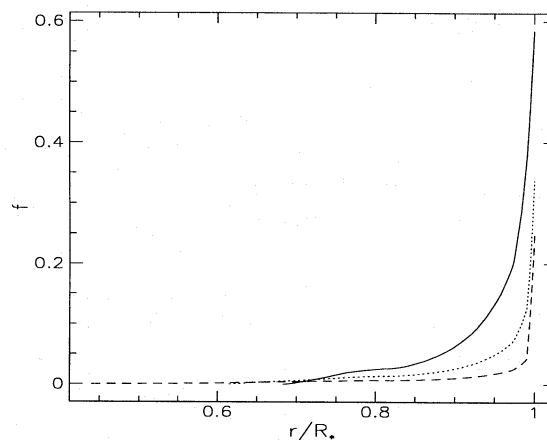


Fig. 4. Effect of Richardson number of convection. Profiles of f for Ri^- equal to -1 (solid line), -10 (dotted line) and -100 (short-dashed line) are shown. Parameters are the same as before with $\mathcal{E}^a = 0.5$.

that feeds the turbulence. So, in the transition region between the convective core and the radiative envelope Γ is low, reflecting the large turbulence resulting from the interaction of these two different regimes. However, deep inside the star, the turbulence weakens as we approach an almost adiabatic transport of energy, so that Ri^- should be enhanced too. Figure 4 represents the influence of Ri^- on the accretion profile f .

When this parameter is large, i.e. the regime approach free convection, matter can reach the central regions of the star but for lower values, the efficiency of the mixing process is reduced and matter is stopped before the center.

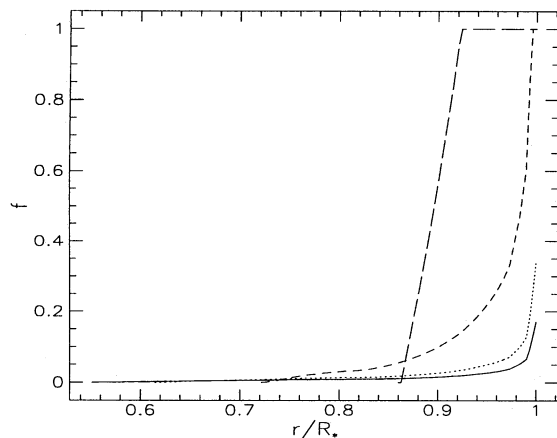


Fig. 5. Profile of f as a function of initial angular momentum. The solid line corresponds to $\xi = 1$, the dotted, short- and long-dashed lines to $\xi = 0.5, 0.1$ and 0.01 , respectively. Parameters are the same as before with $Ri^- = -10$ and $M_{acc} = 10^{29}$ g.

3.2.5. The influence of angular momentum

Our ignorance of the physics in the boundary layer leads us to introduce a new parameter ξ that measures the angular momentum arriving at the stellar surface [see Eq. (12)]. In the formalism here developed, mixing is due to exchange of angular momentum between the accreted matter and the non-rotating stellar material.

A reduction of ξ decreases the efficiency of mixing with depth and the accreted matter tends to accumulate in the surface layers, as shown in Fig. 5. This accumulation leads to a saturation of the penetration function when the critical value 1 is reached. In this case, the stellar surface layers corotate with the accreting matter, thereby terminating the shear mixing. However matter continues to accrete, being partially supported by centrifugal forces.

3.2.6. The influence of geometry

To integrate the departure of spherical symmetry, we have introduced a new parameter ϵ whose effect is to determine the width of the accretion belt through Eq. (21). The case $\epsilon = 0$ refers to the spherically symmetric case and its influence on f is depicted in Fig. 6.

As expected, when the size of the boundary layer reduces, mass is forced to go deeper inside the star to satisfy Eq. (14). But we can see that this effect is weak. Probably the large convective region redistributes the accreted matter over the entirely star and in any case, we will retain that ϵ has a small influence in our one dimensional simulation.

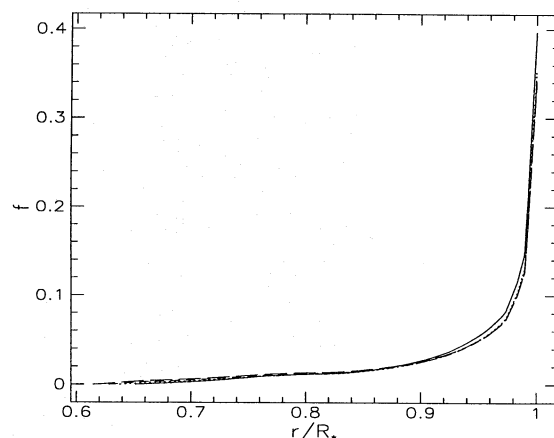


Fig. 6. f as a function of ϵ . The solid solid line corresponds to $\epsilon = 0$ ($\phi_H = 90^\circ$), the dotted, short- and long-dashed lines to ϵ (ϕ_H) equal 0.1 (41.4°), 0.3 (30.6°) and 0.5 (23.5°), respectively. Parameters are the same as before with $\xi = 0.5$ and $M_{acc} = 10^{29}$ g.

4. Discussion and conclusion

Assuming accreted matter is marginally stable in the radiative zone characterized by a Richardson number equal to $1/4$ and generalizing this idea to unstable convective regions with Ri becoming negative, we are able to determine the profile of mass deposition in an accreting star.

All the curves presented in the last section, reveal the relative influence of the different parameters. We first note that geometry or chemical composition have little influence compared to Pe^a , Ri^- or ξ . These parameters produce the maximum changes but the general shape of the penetration function f do not seem to be greatly affected. However, the tendency of an increasing f in the surface leads sometimes to a saturation of this function when reaching the critical value 1. These effects can also be shown through the mass distribution profiles. In Fig. 7 we have plotted the effect of varying parameters on the profile of accreted mass with respect to the mass distribution of the stellar model. We note that when the stabilizing effect of the buoyancy force is reduced, by increasing Ri^- or decreasing Pe^a , the accreted mass profile matches the stellar one. In these situations, convection dominates and matter is distributed more uniformly.

The value $1/4$ used in a radiative zone is the lowest one for stability and its actual magnitude can be arbitrary higher. However its influence is very small because it affects only a very small region of the star. The effect of this parameter is to steepen the function f in this region, and the larger Ri , the more abruptly accreted matter is stopped by the radiative barrier. More crucial is the parameter Pe^a that probably covers a wide range going from 10^{-3} in the external layers to 10^9 in the deep interior, like the convective efficiency Γ does. The uncertainties remaining on this parameter, essentially due to our ignorance of stellar viscosity and thermal properties of the accreted material

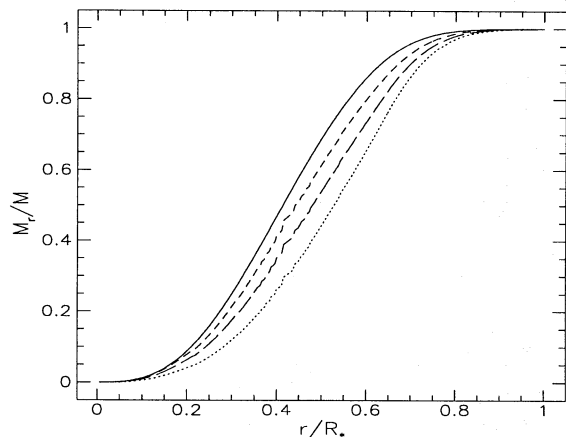


Fig. 7. Normalized profiles of accreted mass compared to the stellar mass distribution (solid line). The dotted line corresponds to the model coming from Fig. 2 with $Ri^- = -10$ and $\mathcal{E}^a = 0.5$. The short- and long-dashed lines correspond to $Ri^- = -100$, $\mathcal{E}^a = 0.5$ and $Ri^- = -10$, $\mathcal{E}^a = 0.2$, respectively. The profiles are normalized either to the stellar mass (solid line) or to the accreted mass, $M_{acc} = 10^{30}$ g.

leads us to keep Pe^a or \mathcal{E}^a as a free parameter. The constant (and high) value of Pe^a taken through all the star is wrong in the outermost layers, but these layers have a very limited extension and do not produce important changes for completely convective stars. The same is true for Ri^- .

Despite relevant progress in angular momentum transport (Spiegel & Zahn 1992 or Zahn 1992) many questions remain opened in that field and our simple model do not pretend to reproduce the distribution of angular momentum inside the star. Indeed our simplifying hypothesis assume angular momentum conservation and do never deal with processes such as loss or redistribution by diffusion. However, the total amount of accreted angular momentum, coupled to stellar evolution computations will be a relevant quantity in order to follow the evolution of the star's total angular momentum or surface rotational velocity history.

Beyond these considerations, a more general question arises: how rotation interacts with convection? Observations of the sun indicate latitudinal differential rotation within the convective zone and solid rotation in the radiative interior (Brown & Morrow 1987) and the same feature seems also occurring in an other solar-type star (Donahue & Baliunas 1992). But does this feature generalize the interaction or is it only applicable for main sequence stars with very thin (in mass) convective zones? To estimate their relative strength, we compare two relevant time scales: the convective time scale $\tau_{conv} \sim H_p/v_{conv}$ where v_{conv} is the convective velocity and the shear time scale $\tau_{sh} \sim (dV/dr)^{-1} \sim \omega^{-1}$ that also represents the mixing time scale for the accreted matter. As Ri^- plays a crucial role, we propose to illustrate its impact through two relevant situations (Fig. 8).

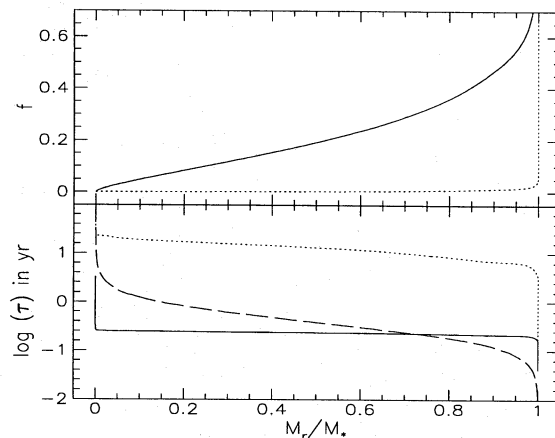


Fig. 8. Characteristic time scales. The solid and dotted lines in the upper panel represent the profile of mass deposition for $Ri^- = -10$ and -10^3 , respectively. Below, the solid and dotted lines correspond to the shear time scale τ_{sh} for the same value of Ri^- as above, i.e. $Ri^- = -10$ and -10^3 , respectively. The long dashed line is the convective time scale τ_{conv} . Fixed parameters are $\dot{M} = 10^{-6} M_{\odot} \text{ yr}^{-1}$, $\xi = 0.3$, $\epsilon = 0.5$ and $\mathcal{E}^a = 0.5$.

First, consider a large value of this parameter: $Ri^- = -10^3$. In such a situation, the graph shows that the shear time scale τ_{sh} exceeds τ_{conv} . Moreover the profile of mass deposition (function f) is very flat, indicating that the distribution of mass in the star remains unchanged. This situation is characteristic of free convection and the fact that $\tau_{sh} > \tau_{conv}$ is due to angular momentum that “brakes” the progression of matter. Conversely, in the case of low value of $Ri^- = -10$, it appears that $\tau_{sh} < \tau_{conv}$ in a large part of the star. Furthermore the profile of f is no more flat, reflecting the efficiency of shear mixing. So, depending on the Ri^- value, free convection or efficient shear mixing can both be encountered. In conclusion, for a given Ri^- , the thermal power of the accretion process tends to compete free convection and forces the mass deposition not to be flat. This tendency is even more pronounced that Ri^- is small, i.e. that convective motions are influenced by the shear flow.

In his paper, MacDonald (1983) found that mixing in the vertical direction of radiative zones operates on thermal time scale. He thus obtains results that, except for the presence of centrifugal forces, approach the case of radial accretion. But depending on the mixing time scale we are confronted with two extreme cases: (i) the accretion time scale $\tau_{acc} \sim M_{acc}/\dot{M}$, is long compared to the mixing time scale and accretion is well described by the Richardson criterion or (ii) conversely the mixing time scale is longer; accretion is radial and there is no mixing in the radial direction as in McDonald's case. Our model applies in the former case because of the shortness of τ_{sh} .

This model is a starting point of an investigation that will follow the evolution of young stellar objects supporting accretion. This formalism will be implemented inside the stellar evolution

code developed by Forestini (1991, or for a more recent review Forestini 1994). In this context, at each time step we will run the accretion procedure, leading to a profile of mass and energy deposition, afterwards the code will arrange the structure and the iteration will be repeated. In a next paper we will also describe the energetics that is attendant on the accretion process. Indeed various energy sources originate from different processes acting during accretion. The transfer of angular momentum is accompanied by dissipation caused by the shear. In our treatment angular momentum is conserved but not rotational kinetic energy. Part of this energy given up by the rotational motion goes into driving the turbulent mixing, the remainder is dissipated as heat through viscous processes. Another source of energy comes from the internal energy of the accreted matter. If the accreted globule is not forced to move adiabatically, during its displacement it will release or absorb heat as it is sinking or rising. Moreover, when entering the star, matter must release its excess of angular momentum from its Keplerian value to ξj_K . This reduction is fully dissipative and can be integrated as a boundary condition at the stellar surface.

Acknowledgements. It is a great pleasure to thank Jean Paul Zahn for critically reading the manuscript and for stimulating discussions that enlightened the content of this paper.

References

- Böhm-Vitense, E., 1958, Z.Astrophys. 6, 487
 Brown, T.M., Morrow, C.A. 1987, ApJ, 314, L21
 Chandrasekhar, S., 1961, Hydrodynamics and Hydromagnetic Stability
 Oxford University Press, Clarendon Press edition
 Cox, J.P., Guili, R.T., 1968, Principles of stellar structure Gordon and Breach, Science Publishers
 Donahue, R.A., Baliunas, S.L., 1992, ApJ, 393, L63
 Favre, et al., 1976, La Turbulence en Mécanique des Fluides Gauthier-Villars Bordas
 Forestini, M., 1991, Ph. D.Thesis (Université Libre de Bruxelles, Belgium)
 Forestini, M., 1994, A&A, 285, 473
 Fujimoto, M.Y., Iben, I.Jr., 1989, ApJ, 341, 306
 Fujimoto, M.Y., 1993, ApJ, 419, 768
 Hartmann, L.W., Kenyon, S.J., 1990, ApJ, 349, 190
 Kippenhahn, R., Meyer-Holfmeister, E., 1977, A&A, 54, 539
 Kippenhahn, R., Thomas, H.-C., 1978, A&A, 63, 265
 Knobloch, E., Spruit, H.C., 1982, A&A, 113, 261
 Kutter, G.S., Sparks, W.M., 1987, ApJ, 321, 386
 Kutter, G.S., Sparks, W.M., 1989, ApJ, 340, 985
 Lioure, A., Lecontel, O., 1994, A&A, 285, 185
 Livio, M., Truan, J.W., 1987, ApJ, 318, 316
 MacDonald, J., 1983, ApJ, 273, 289
 MacGregor, K.B., Brenner M., 1991, ApJ, 376, 204
 Maeder, A., 1995, A&A, 299, 84
 Mercer-Smith, J.A., Cameron, A.G.W., Epstein, R.I., 1984, ApJ, 279, 363
 Palla, F., Stahler, S.W., 1988, ApJ, 332, 804
 Palla, F., Stahler, S.W., 1992, ApJ, 392, 667
 Regev, O., 1983, A&A, 126, 146
 Regev, O., Houterat, A.A., 1988, MNRAS, 232, 81
 Schatzman, E., Praderie, F., 1990, Les Etoiles InterEditions, Editions du CNRS
 Shaviv, G., Starrfield, S., 1987, ApJ, 321, L51
 Shaviv, G., Starrfield, S., 1988, ApJ, 335, 383
 Sparks, W.M., Kutter, G.S., 1987, ApJ, 321, 394
 Spiegel E.A., Zahn J.-P., 1992 A&A, 265, 106
 Stahler, S.W., Shu, F.H Taam, R.E, 1980, ApJ, 241, 637
 Sung, C-H. 1974, A&A, 33, 99
 Townsend, A.A. 1958, J.Fluid Mech. 4, 361
 Zahn, J.P. 1974, Rotational Instabilities and Stellar Evolution. In: Stellar Instabilities and Evolution, ed. Ledoux et al., Reidel Publ. Co., p.185
 Zahn, J.P. 1983, Astrophysical Processes in Upper Main Sequence Stars (13 th. Advanced Course of Saas Fee, Publisher: Geneva Observatory)
 Zahn, J.P. 1992, A&A, 165, 115

This article was processed by the author using Springer-Verlag L^AT_EX A&A style file version 3.