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ON THE ROTATIONAL EVOLUTION OF YOUNG LOW-MASS STARS

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ABSTRACT

Observations of young clusters indicate that a significant fraction of solar-type stars are rotating very slowly, with equatorial velocities less than 10 km s^{-1}. So far, models have failed to reproduce a sufficiently large proportion of these stars on the zero-age main sequence. On the basis of the idea that the mixing length in convection theories could depend on the size of the convective zone (Canuto & Mazzitelli), we examine the influence of a varying mixing-length parameter α on the rotational evolution of solar-type stars. A decreasing α (owing to evolution) in the mixing-length theory (MLT) leads to a slower contraction rate and to a larger stellar moment of inertia. The stellar spin-up is consequently reduced, and this helps to increase the number of very slow rotators present in young clusters. We also investigate the possibility that α could depend on the rotation rate, and show the consequences of this parameterization for the lithium surface abundance.

Subject headings: convection — stars: evolution — stars: interiors — stars: late-type — stars: rotation

1. INTRODUCTION

The rotational evolution of solar-type stars is now relatively well understood. Slowly rotating T Tauri stars (with an age less than ~10 Myr) can be explained by invoking a star-disk coupling (Königl 1991). In this scheme, the magnetic field lines arising from the stellar magnetosphere thread the disk out of the corotation radius and give rise to a braking torque. An equilibrium configuration, in which the accelerating torque owing to mass accretion balances the braking torque induced by the magnetic field, is then reached. The star's rotation rate is then locked at a constant angular velocity. Several computations (e.g., Cameron & Campbell 1993; Armitage & Clarke 1996) indicate that for accretion rates of the order of 10^{-9} to 10^{-11} $M_\odot$ yr^{-1}, it is possible to balance the spin-up torque provided that the magnetic field strengths are of the order of a few hundred gauss.

Such magnetic fields have been measured by Zeeman splitting (e.g., Johnstone & Penston 1986, 1987) or estimated on the basis of the intense starspot activity often exhibited by these objects (e.g., Donati et al. 1992). Another alternative to account for these slow rotators was proposed by Tout & Pringle (1992). In their model, the differentially rotating fully convective star generates a magnetic dynamo that deposits a significant amount of energy at the surface of the star, which in turn leads to substantial mass loss ($\dot{M}_{\rm wind} \approx 10^{-7} M_\odot$ yr^{-1}). The magnetically coupled stellar winds emanating from these stars are responsible for the loss of angular momentum and subsequent spin-down.

Observations of post–T Tauri stars (PTTSs) with ages of 10–40 Myr from Bouvier et al. (1997a) show that the rotational velocity distribution of T Tauri stars evolves to higher velocities as a consequence of structural evolution. It is worth noting that at this age, 30% of PTTSs are slow rotators, with rotational velocities of less than 20 km s^{-1}. At the age of α Per (50 Myr), observations show a considerable spread in the rotation rates (Prosser 1992). Roughly one-half of the solar mass stars are rapid rotators (with equatorial velocities $50 \lesssim v \sin i \lesssim 200$ km s^{-1}), while many of the rest rotate rather slowly with rotational velocities below 20 km s^{-1}. In the Pleiades (80 Myr), the fraction of rapid rotators has considerably decreased and slow rotators ($v \sin i \lesssim 20$ km s^{-1}) represent now about 80% of the population. Within only the 30 Myr separating the ages of these two clusters, observations by Prosser (1992) and Soderblom et al. (1993b) show a 50% spin-down of the most rapidly rotating stars. Finally, in the Hyades (600 Myr) all stars have equatorial velocities typically below 10 km s^{-1}.

These observations indicate that just after their arrival on the zero-age main sequence (ZAMS), stars undergo an efficient braking. The braking mechanism that is the origin of this rapid spin-down is supposed to be the action of a magnetically coupled wind (Weber & Davis 1967). A scenario to explain the global evolution can now be drawn. During the T Tauri phase stars are efficiently braked by the presence of the disk and/or significant mass loss. Once the influence of the disk recedes, the star spins up as a result of its contraction. During this short period that brings the star onto the ZAMS (between 10 and 50 Myr), little angular momentum is extracted from the star because the spin-down timescale for angular momentum loss owing to the magnetized stellar wind, τ_w, remains large, of the order of or larger than the Kelvin-Helmholtz timescale $\tau_{\rm KH}$. However, once the star reaches the ZAMS, braking by a magnetically coupled wind efficiently produces a rapid deceleration.

To further increase the braking efficiency, it was suggested (Endal & Sofia 1981) that angular momentum is removed preferentially from the surface layers of the star, i.e., from the convective envelope. This core-envelope decoupling accelerates the spin-down of fast rotators on the ZAMS and can account for the 50% spin-down between α Per and the Pleiades (Soderblom et al. 1993c; Keppens, MacGregor, & Charbonneau 1995). Note that Bouvier, Forestini, & Allain (1997b, hereafter BFA) achieve similar results with a solid-body rotation model by invoking a distribution of disk lifetimes. Stars with short-lived disks can spin up to velocities as high as 200 km s^{-1}, while long-lived disks can account for the large fraction of slow rotators. However, their models require slightly longer star-disk coupling, with 20% of the stars still having a disk at an age of 10 Myr.

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In spite of these general successes in explaining the observations, all the models suggested so far (Soderblom et al. 1993c; Keppens et al. 1995; BFA) still fail to reproduce the proportion of very slow rotators (VSRs) with $v \sin i \lesssim 10$ km s^{-1} observed in young clusters. Observations suggest a progressive increase in the fraction of VSRs: between 10 and 40 Myr, 15% of the PTTSs have equatorial velocities less than 10 km s^{-1}; this fraction is of the order of 25% in the α Per cluster and it reaches 50% at the age of the Pleiades (Bouvier et al. 1997a). The problem is to account for the increasing fraction of VSRs during the given time. The problem is exacerbated by the fact that between 10 and 50 Myr the star experiences a large contraction and important structural changes that act to increase its rotation rate. One potential solution to the latter problem is to invoke a long star-disk coupling period. For certain parameters, the stellar rotation rate does not increase too much, but it requires rather long disk lifetimes ($\sim$10 Myr), which may or may not be realistic.

The failure of the models to reproduce the VSRs leads us to the conclusion that other physical processes in addition to star-disk locking and wind braking operate during the pre–main-sequence (PMS) phase. During this evolutionary phase, the star undergoes significant changes with the passage from a fully convective structure to a radiative one. In the present work, we investigate how a change in the convective parameter of the mixing-length theory, α, affects the resulting rotation rates. Indeed, stellar quantities governing the rotational evolution of stars (radius, depth of the convective envelope, moment of inertia) are strongly dependent on the value of this parameter. A smaller value of α influences the stellar structure in such a way that the star arrives on the ZAMS with a larger moment of inertia, radius, and radiative core. We thus expect that modifications in α will influence the angular momentum evolution of young stars.

In § 2 we describe how the mixing-length parameter could evolve and present briefly the equations describing the angular momentum evolution of solar-type stars. In § 3 we present and discuss our results, and a summary and conclusions follow.

2. A NEW PHYSICAL PROCESS IN THE ROTATIONAL EVOLUTION

2.1. *The Mixing-Length Parameter α*

Modeling turbulent stellar convection has attracted considerable attention during the last decade. The standard mixing-length theory (MLT; Böhm-Vitense 1958) has proved to be a very valuable tool, simple in its formulation and successful in its application. However, this theory suffers from two well-known problems. First, it is assumed that a convective element, after traveling a distance (mixing length) Λ, dissolves and releases its excess heat. Unfortunately, MLT cannot determine the value of the mixing length, and it is forced to make the assumption that Λ is proportional to the local pressure scale height H_p, writing

$$\Lambda = \alpha H_p \; , \tag{1}$$

where the constant of proportionality α is the mixing-length parameter. Second, MLT assumes that each convective element has the same size Λ. In an attempt to resolve these two main limitations, Canuto & Mazzitelli (1991, hereafter CM) developed a new model of convection that (1) accounts for the full spectrum of turbulent eddies rather than for only one eddy size, as done in MLT, and (2) uses a new expression for the mixing length, free of adjustable parameters,

$$\Lambda = z \; , \tag{2}$$

where z is the distance to the top of the convection zone. This expression accounts for the fact that thermally driven turbulence is a nonlocal phenomenon. With this new theory, CM were able to fit the Sun's data (surface temperature, luminosity, thickness of the convective envelope), without adjustable parameters, to within $\sim$0.2% accuracy.

Equation (2) shows that the mixing length is correlated with the size of the convective envelope. A formal comparison between this expression and equation (1) leads to the idea that, if we still want to work in the framework of MLT, the value of α must vary as the depth of the convective zone during stellar evolution. A larger value of this parameter may be expected in the early evolution because the star is fully convective and/or the gravitational energy production is uniformly distributed inside the structure. The shrinking of the convective envelope with the development of the radiative core would subsequently decrease the value of this parameter and eventually, when the star reaches the ZAMS, α would remain constant.

Several authors have discussed the appropriateness of using a single value of α to model a wide variety of stars or stars in different evolutionary phases (e.g., Edmonds et al. 1992; Lydon, Fox, & Sofia 1993), but so far no real consensus has emerged. Ordinary stellar structure calculations typically use a global value of α throughout the entire convective region, with values ranging from 0.5 to 3. However, Canuto (1989, 1990) has pointed out that, if one attempts to account for anisotropic effects in MLT, α must be made a function of the thermodynamic quantities. Using this new formalism, Kiziloglu & Civelek (1992) showed that very high values of α, up to 200 (see their Fig. 3), can be reached in solar-type envelopes. This dependence of α on the depth of the convective envelope is also observed when modeling convection at the surface of white dwarfs. Comparisons of hydrodynamic simulations of convection in the surface layer of these stars with MLT (e.g., Ludwig, Jordan, & Steffen 1994) indicate that α can reach values as high as 4 or 5 at the base of the thin convective envelope. Adding to the uncertainties surrounding the determination of this parameter, Sackmann & Boothroyd (1991) noted that α is very sensitive to the input physics. While modeling AGB stars and changing their opacity tables, they had to increase their mixing-length parameter "dramatically," namely, from 2.5 to 4.1, in order to obtain consistency with previous models.

To summarize the present situation, the determination and evolution of the mixing-length parameter are highly uncertain. In practice, this parameter is actually adjusted so as to obtain the solar radius at the age of the Sun, and stellar modeling indicates that reasonable values of α range from 1 to 4. Inspired by the work of CM, we adopt the following parameterizations.

We define a function $\alpha(t)$ as equal to

$$\alpha(t) = aX(t) + b \; , \tag{3}$$

where a and b are normalization constants. The function $X(t)$ is defined by

$$\text{case A:} \quad X(t) = R_{\mathrm{conv}}(t)/R_{\odot} \; ,$$

$$\text{case B:} \quad X(t) = R_{\mathrm{conv}}(t)/R \; ,$$

where R and R_{conv} are the radii of the star and of the convective zone, respectively. The second expression (case B) corresponds to a constant value of α as long as the star is fully convective and for a rapid drop as the radiative core develops. Note that these relations imply that α is constant in the convective regions.

2.2. *The Rotational Evolution of the Star*

To follow the rotational evolution of young stars, we adopt the model presented by BFA in which we include in addition the core-envelope decoupling (MacGregor & Brenner 1991, hereafter MB). Briefly, our model is as follows:

1. For $t < t_{disk}$, the star is coupled to the disk, and its surface rotation rate Ω is maintained constant $\Omega = \Omega_{init}$.
2. When the disk coupling vanishes ($t > t_{disk}$), magnetic braking owing to a stellar wind takes place. We use the parametric formulation of Kawaler (1988),

$$\frac{dJ}{dt} \sim \Omega^{1+4an/3} R^{2-n} \dot{M}^{1-2n/3} M^{-n/3} , \tag{4}$$

where J is the angular momentum, $\dot{M}$ is the wind mass-loss rate, and n describes the topology of the magnetic field ($B \propto r^{-n}$) and is arbitrarily taken to be constant, $n = \frac{3}{2}$. This corresponds to a magnetic configuration that is intermediate between a radial and a bipolar field. We note that in a recent work (Solanki, Motamen, & Keppens 1997) it has been suggested that the latitudinal concentration of magnetic flux may be influenced by the star's rotation; this would effectively be equivalent to a time dependence of n. The parameter a describes the dependence of the magnetic field on the rotational velocity Ω ($B \propto \Omega^a$). Here we consider two cases: if the rotation rate is lower than a critical value ω_{sat}, which is a free parameter, the dynamo-generated magnetic field is proportional to Ω ($a = 1$) and

$$\left(\frac{dJ}{dt}\right)_w = -K\Omega^3 \left(\frac{R}{R_\odot}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{-1/2} , \tag{5}$$

while if $\omega \geq \omega_{sat}$, the magnetic field saturates ($a = 0$) and

$$\left(\frac{dJ}{dt}\right)_w = -K\Omega\omega_{sat}^2 \left(\frac{R}{R_\odot}\right)^{1/2} \left(\frac{M}{M_\odot}\right) , \tag{6}$$

where K is a calibration constant taken to be 2.7×10^{47} g s cm^2 (BFA). Note that equation (5) is equivalent to the Skumanich (1972) relationship, $\Omega(t) \propto t^{-1/2}$, observed for the slow rotators on the main sequence.

3. We include a core-envelope decoupling that assumes that the radiative core and the convective envelope have different rotation rates. MB's model assumes that an amount of angular momentum ΔJ is transferred from the core to the envelope on a timescale τ_c (see MB for the expression of ΔJ). The evolution of the core and envelope angular momenta (J_{core} and J_{conv}, respectively) can then be derived, and they lead to the following equations (MacGregor 1991):

$$\frac{dJ_{core}}{dt} = -\frac{\Delta J}{\tau_c} + \frac{2}{3}\, \Omega_{conv} R_{core}^2 \frac{dM_{core}}{dt} , \tag{7}$$

$$\frac{dJ_{conv}}{dt} = \frac{\Delta J}{\tau_c} - \frac{2}{3}\, \Omega_{conv} R_{core}^2 \frac{dM_{core}}{dt} - \frac{J_{conv}}{\tau_j} , \tag{8}$$

where $\tau_j = J_{conv}/(dJ/dt)_w$ is the e-folding time for the wind-induced angular momentum loss from the convective zone and dM_{core}/dt is the mass exchange rate at the boundary R_{core} between the core and the convective envelope.

3. RESULTS AND DISCUSSION

3.1. *Stellar Evolution with Varying* α

The stellar evolution code used in these computations is presented in Siess, Forestini, & Dougados (1997). We have incorporated in the code the new prescriptions for the mixing-length parameter (cases A and B) and computed the new stellar structure with varying α. The code then allows for the determination of the moments of inertia of the core and envelope, $I_{core}(t)$ and $I_{conv}(t)$, as well as the changing core radius R_{core} and the mass exchange rate dM_{core}/dt. As an illustration, we depict in Figure 1 the effects of using different (constant) values for the mixing-length parameter α on some relevant stellar quantities.

As α increases, energy can be carried out by the convective elements over a longer path. The convective flux is consequently enhanced and, in order to equilibrate the energy losses at the surface, the star contracts more efficiently. As a result, the moments of inertia of both the convective envelope and the radiative core decrease. However, it is worth noting that a larger value of α (1) hastens the development of the radiative core (a star with a larger α has a higher central temperature and thus a lower central opacity and radiative gradient), (2) increases the evolutionary timescale τ_{KH} as a result of a smaller radius and luminosity, and (3) leads to a relatively thicker convective envelope on the ZAMS, which corresponds to a larger I_{conv}. As we will see, these effects conspire to ensure a stronger braking of the star. Indeed, under the assumption of core-envelope decoupling and assuming α is a decreasing function of time, (1) an earlier braking of the convective envelope is favored as it appears sooner (this is, however, a small effect); (2) the wind braking timescale τ_w becomes shorter earlier in the evolution, and thus the braking laws act more efficiently earlier; and (3) the convective envelope is braked more rapidly as its moment of inertia becomes smaller.

3.2. *Parameters*

Keppens et al. (1995) and BFA have examined the effects of using different values for the parameters in the treatment of core-envelope decoupling and solid rotation. Here we adopt values of the parameters that were found to be reasonable by the previous authors; we assume an initial rotation period of 8 days for T Tauri stars, which corresponds to $\Omega_{init} = 3\ \Omega_\odot$, as suggested by observations (Bouvier et al. 1993; Edwards et al. 1993). We assume $\omega_{sat} = 10\ \Omega_\odot$, several values for the disk lifetime $t_{disk} = 7 \times 10^5$, 4×10^6, and 10^7 yr and a core-envelope coupling timescale $\tau_c = 10$ Myr. As noticed by Keppens et al. (1995), the choice of these parameters is severely constrained. On one hand, τ_c must be short enough so that the angular momentum transferred from the core to the envelope will allow the PMS star to become a fast rotator. On the other hand, τ_c must be sufficiently long to reproduce the significant spin-down of the envelope on the ZAMS. The dynamo saturation is required to account for the large spread in the rotation rates, and the level of saturation ω_{sat} is determined so as to reproduce the spin-down to slow rotators by the Hyades age. The value of

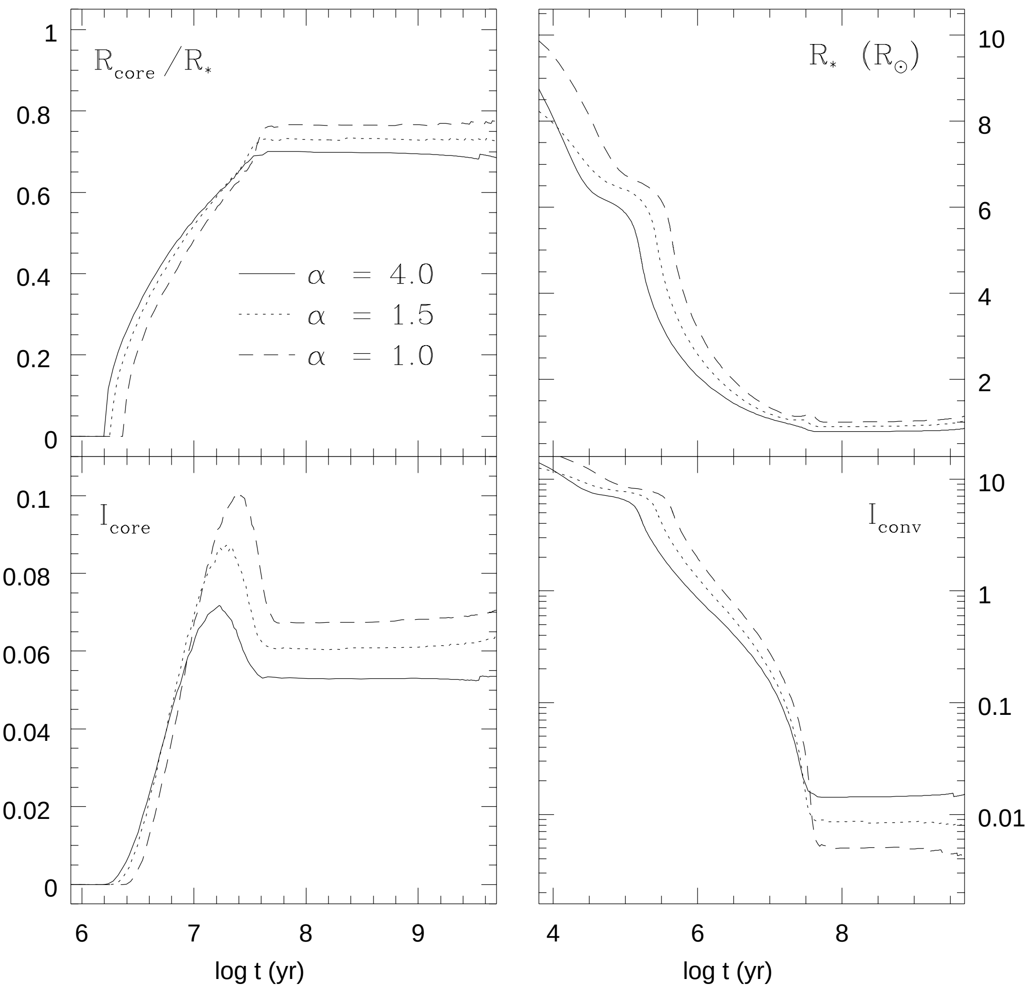

FIG. 1.—Influence of α on the evolution of stellar quantities. *Upper-left panel:* Evolution of the boundary radius $R_{\rm core}$ between the radiative core and convective envelope. *Upper-right panel:* Evolution of the stellar radius. *Lower panels:* Time evolution of the moments of inertia of the core, $I_{\rm core}$ (*left*), and envelope, $I_{\rm conv}$ (*right*). These characteristics are for a 1 $M_\odot$ star, and the values of α are assumed constant throughout the evolution.

$\omega_{\rm sat}$ depends on the adopted braking law and is in the range 10–20 $\Omega_\odot$. Finally, in cases A and B we allow α to cover the range 1–4. The constants a and b are determined by the requirement to fit the solar radius at the age of the Sun.

3.3. *Rotational Evolution*

Figure 2 depicts the effect of α on the rotational evolution of solar-type stars in the case of core-envelope decoupling and solid rotation. For comparison, we have also plotted the evolution of the angular velocity of a standard track, where α is constant throughout the computations ($\alpha = 1.5$, case C). Our calculations reveal that, when the mixing length decreases during the evolution, the star rotates more slowly than in a standard case (α constant). Indeed, with a larger initial value of α, the moment of inertia of the star is lower and, given an initial rotation rate ($\Omega_{\rm init}$), one obtains a lower angular momentum. So, when the stars decouple from the disk, those with a higher angular momentum will spin up faster and reach larger angular velocities. When the star finally arrives on the ZAMS, the evolutionary timescale becomes larger than the wind-braking timescale and the braking law can be applied efficiently. At the age of the Hyades, the stars have spun down considerably and present very similar angular velocities. Finally all the models converge to the Sun's rotation rate at the age of the Sun.

Figure 2 also shows that the rotational evolution of solar-type stars crucially depends on the adopted law for $\alpha(t)$. If this parameter maintains a large value (around 4) for a long period of time, as in case B, then the contraction rate is globally less significant and the rotation rate reaches lower values. Namely, the star is forced to rotate more slowly by imposing a smaller radius and a smaller moment of inertia. Conversely, when $\alpha(t)$ varies continuously, as in case A, the stellar structure evolves into a more distended configuration. The star experiences a stronger contraction and thus spins up more efficiently. We also see that because the value of α is always smaller in case A than in case B, the rotation rate in the latter case is always lower.

Quantitatively, for $t_{\rm disk} = 4 \times 10^6$ yr, at the age of α Per, a star evolving with constant α rotates approximately 10% and 25% faster than in cases A and B, respectively (Fig. 2*a*). If we take a higher value of $\omega_{\rm sat}$, say $\omega_{\rm sat} = 15\ \Omega_\odot$, we find that the differences between case C and cases A and B are now on the order of 5% and 15%, respectively. Thus, we observe that the additional braking induced by changes of $\alpha(t)$ decreases with increasing $\omega_{\rm sat}$. Indeed, if $\omega_{\rm sat}$ is increased, we allow an important angular momentum loss for slow rotators ($dJ/dt \propto \Omega^3$) that minimizes the effect owing to $\alpha(t)$. We also notice that within the core-envelope decoupling model, we obtain a more efficient braking than

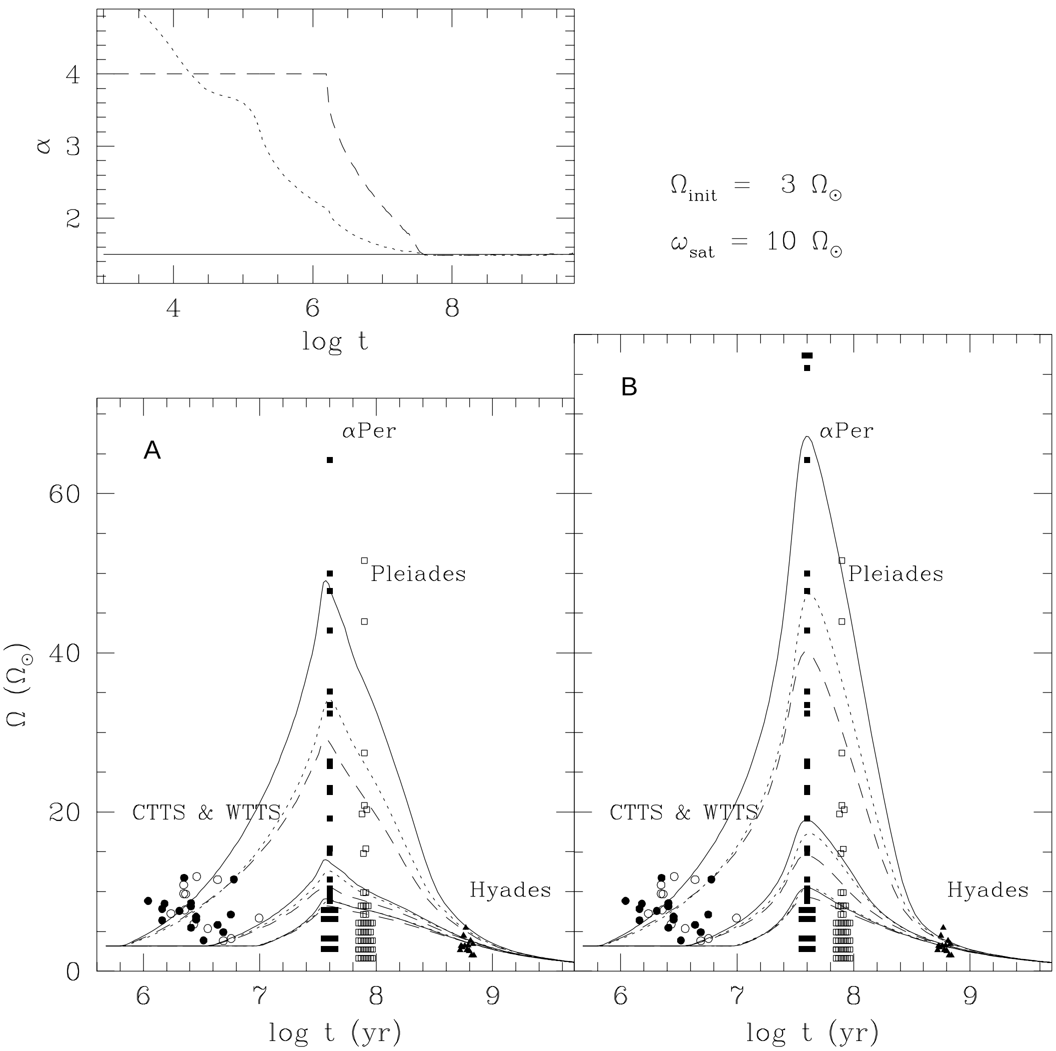

FIG. 2.—*Left panel:* Rotational evolution of solar-type stars in the case of a core-envelope decoupling. *Right panel:* Rotational evolution of a solid-body rotation in the case of a core-envelope decoupling. The solid line corresponds to the solar model with $\alpha = 1.5$ (case C), the dotted and dashed curves to cases A and B, respectively. The behavior of α is presented in the small top panel. Disk lifetimes $t_{\rm disk}$ are equal to 7×10^5, 4×10^6 and 10^7 yr. Data: T Tauri stars (age 1–10 Myr; *filled circles*, classical T Tauri stars [CTTSs]; *open circles*, weak-line T Tauri stars [WTTSs]) come from Bouvier et al. (1993), stars in α Per cluster (Prosser 1992), Pleiades (Soderblom et al. 1993b), and Hyades (Radick et al. 1987).

in the solid-body scheme. From Figure 2*b*, we infer that in cases A and B, the star's rotation rate is slower than in the standard model by approximately 5% and 20%, respectively (compared to the 10% and 25% found previously). Figure 2 also indicates that the difference in angular velocity between the different models decreases with time. By the age of the Pleiades, the rotation rates are very similar between cases A and C, while in case B the star still rotates 15% slower than in the standard model.

Finally, if we change the law for α and allow this parameter to reach lower values (below unity), we observe an even stronger spin-down of the star. At the age of α Per (for $t_{\rm disk} = 4 \times 10^6$ yr and $\omega_{\rm sat} = 10\ \Omega_\odot$), a star with α approaching unity can rotate 35% less rapidly than in the standard scheme.

4. DISCUSSION AND CONCLUSION

An examination of the rotational evolution of young stars has allowed us to suggest that, in the context of MLT, the parameter describing the mixing length α may change during the evolution. Our calculations have shown that taking into account an evolution of α can produce a more efficient spin-down of solar-type stars by the ages of α Per and the Pleiades. This result may help in solving the problem of very slow rotators and in relaxing the hypothesis of excessively long disk lifetimes ($t_{\rm disk} \gtrsim 10$ Myr). Indeed, this new effect accounts for an additional braking of the order of 10%–30%, depending on the parameters. We also found that larger values of $\omega_{\rm sat}$ or $t_{\rm disk}$ reduce the differences between models with different prescriptions for α. Note, however, that rapid rotators can still be reproduced if the coupling timescale between the star and the disk is very short. For example, if we adopt a disk lifetime of 2×10^5 yr, we obtain rotational velocities in cases A and B on the order of 190 and 170 km s^{-1}, respectively.

In addition, we find that the rotational evolution of solar-type stars depends significantly on the form of the time dependence of α. Specifically, if this parameter is maintained at a high value for a long time and then decreased rapidly with the development of the radiative core (case B), a more significant spin-down is produced than if it is decreased continuously.

We can now attempt to deduce the proper parameterization required for the observations. We found that a

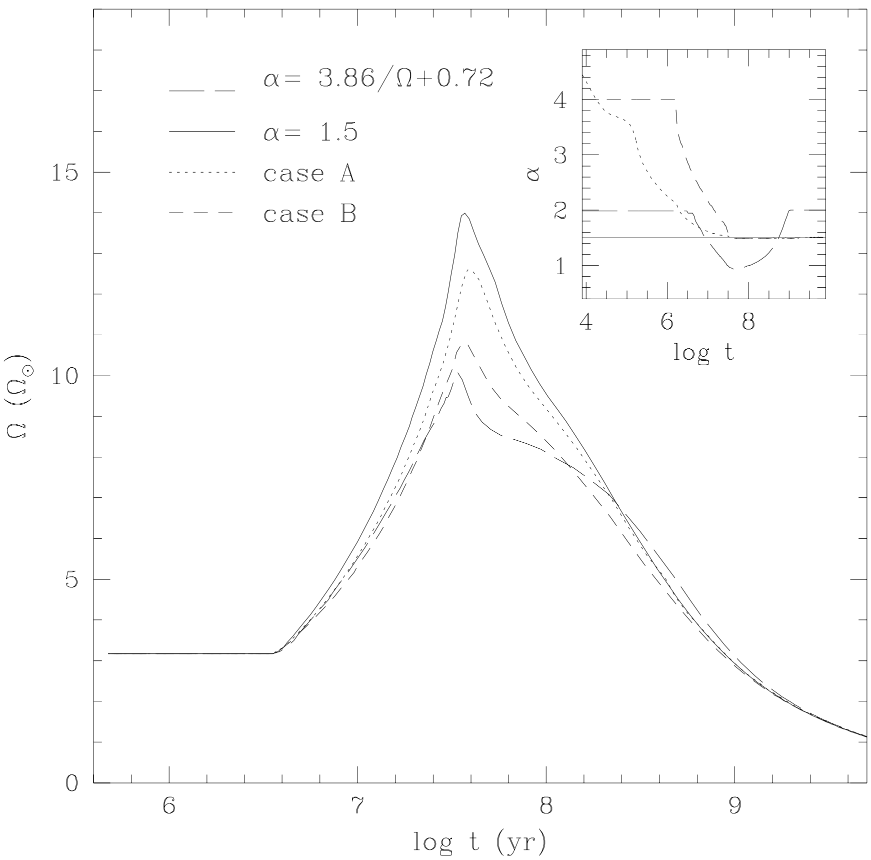

FIG. 3.—Influence of different parameterizations of α on the rotational evolution of solar-type stars. The disk lifetime is $t_{\rm disk} = 4 \times 10^6$ yr and the level of saturation $\omega_{\rm sat} = 10\ \Omega_\odot$. The solid line corresponds to the solar model with $\alpha = 1.5$, the dotted and short- and long-dashed curves to different laws for α as indicated in the inset. We notice the strong braking of the star when $\alpha \propto \Omega^{-1}$ (see text).

decrease in the value of α tends to produce more slow rotators. A potential scenario is one in which the parameterization of α depends on the rotation rate itself. Indeed, high rotational velocities can twist the convective cells, which results in a shortening of the mixing length. In a scheme of this type, α is a decreasing function of Ω. Thus, the star could maintain a large value of α during the T Tauri phase when the star is locked to the disk. The rapid spin-up following the star-disk decoupling would reduce the value of α and, thus, induce a more efficient braking of the surface layers. Thereafter, during the spin-down toward the main sequence, α increases again in order to fit the solar structure at the age of the Sun.

Figure 3 demonstrates the effects of such a scheme when α scales as Ω^{-1}. We assume an ad hoc law of the form

$$\alpha = c\left(\frac{\Omega}{\Omega_\odot}\right)^{-1} + d\ , \tag{9}$$

where the parameters c and d are defined in such a way that α is equal to 2 for $\Omega = \Omega_{\rm init}$, and at the maximum rotation rate ($\Omega \simeq 45\ \Omega_\odot$) α cannot decrease below 0.8. Furthermore, we restrict α not to exceed 2 on the main sequence. As we can see from Figure 3, this new parameterization induces a more efficient spin-down of the star, and thus a closer agreement to observations.

Finally, we would like to point out that this prescription for α may explain observations of lithium abundances in solar-type stars. As we show in Figure 4, faster rotators, i.e.,

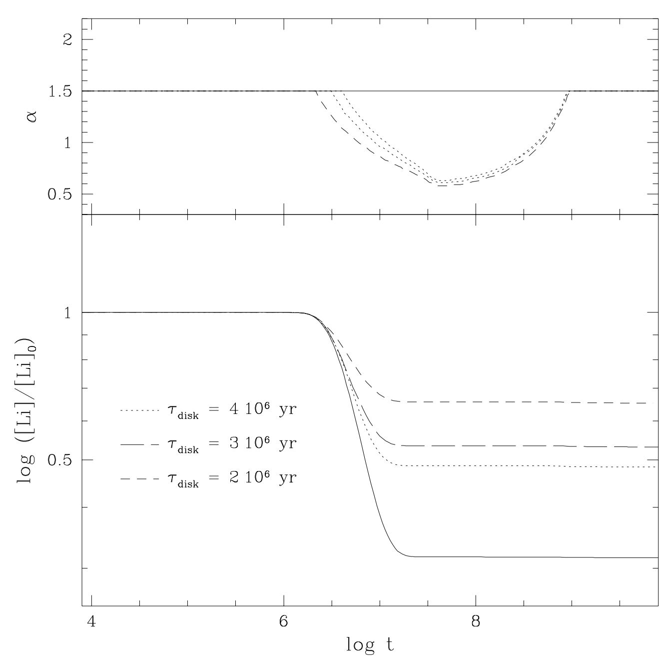

FIG. 4.—Influence of different parameterizations of α on the ^{7}Li surface abundance. The solid line corresponds to the solar model with $\alpha = 1.5$; the dotted and long- and short-dashed lines follow the same law for α given by eq. (9) but different disk lifetimes equal to 2×10^6, 3×10^6 and 4×10^6 yr, respectively. The profiles of α are shown in the upper panel. We notice that the fastest rotators, i.e., those with the shortest $\tau_{\rm disk}$, have the highest ^{7}Li surface abundance.

those that decouple from the disk earlier, deplete their lithium less efficiently. Indeed, when α decreases, the convective envelope shrinks. Lithium is then transported less deeply into the stellar interior and consequently less lithium is destroyed. This result is consistent with numerous observations (e.g., Soderblom et al. 1993a; De Medeiros, Do Nascimento, & Mayor 1997) that indicate a strong correlation between the rotational velocity of single stars and their Li abundance.

Concerning later evolutionary stages, observations of the rotational velocities of evolved stars (e.g., Gray & Nadar 1985; Gray 1989; De Medeiros & Mayor 1991) show a sudden transition from rapid to slow rotation occurring at spectral types G0 III and F8 IV in class III and IV giants, respectively. As noticed already by Charbonneau (1992), this cutoff occurs at essentially the same spectral types as the one observed in main-sequence stars, indicating that similar braking mechanisms (magnetic) may also apply to post–main-sequence stars. During this evolutionary phase, the moment of inertia of the star increases owing to the expansion of the outer layers and a deep convective envelope develops. As a result of these structural changes, the rotation rate decreases. However, if we keep the prescription that the mixing length is proportional to the depth of the convective envelope (cases A and B), we can expect an even more efficient braking during this period. Indeed, the increase in α (resulting from the deepening of the convection layer) will lengthen the stellar evolutionary timescale and increase the moment of inertia of the convective envelope (Fig. 1). These effects will result in an efficient braking, which is consistent with observations.

Finally, it becomes more and more clear that an ultimate treatment of rotation and its effects on convection will require multidimensional calculations of the stellar structure and evolution.

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