Hydrodynamic Simulations of Shell Convection in Stellar Cores

Miroslav Mocák, Ewald Müller, and Lionel Siess

Abstract Shell convection driven by nuclear burning in a stellar core is a common hydrodynamic event in the evolution of many types of stars. We encounter and simulate this convection (1) in the helium core of a low-mass red giant during core helium flash leading to a dredge-down of protons across an entropy barrier, (2) in a carbon-oxygen core of an intermediate-mass star during core carbon flash, and (3) in the oxygen and carbon burning shell above the silicon-sulfur rich core of a massive star prior to supernova explosion. Our results, which were obtained with the hydrodynamics code HERAKLES, suggest that both entropy gradients and entropy barriers are less important for stellar structure than commonly assumed. Our simulations further reveal a new dynamic mixing process operating below the base of shell convection zones.

1 Introduction

Our knowledge of stellar core convection stems from one-dimensional stellar evolutionary calculations assuming a hydrostatic stellar structure and describing dynamic processes (like, e.g., convection) by local and linear theories (Weiss et al. 2004). This approach is computationally feasible and predicts observables. However, it involves some degeneracy due to necessary utilization of observationally constrained free parameters (Montalbán et al. 2004), which may vary during the star's evolution, and from star to star. One way to check the 1D results is by means

M. Mocák (✉) · L. Siess
Institute of Astronomy and Astrophysics at the Université Libre de Bruxelles, 1050 Bruxelles, Belgium
e-mail: mmocak@ulb.ac.be; siess@astro.ulb.ac.be

E. Müller
Max-Planck-Institut für Astrophysik, 85748 Garching bei München, Germany
e-mail: ewald@mpa-garching.mpg.de

A. Miglio et al. (eds.), *Red Giants as Probes of the Structure and Evolution of the Milky Way*, Astrophysics and Space Science Proceedings,
DOI 10.1007/978-3-642-18418-5_9,

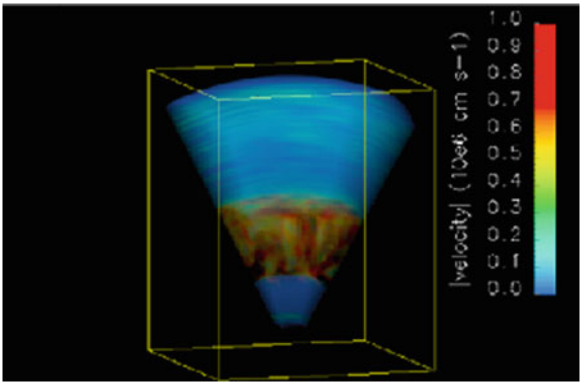

Fig. 1 3D volume rendering of the velocity amplitude of shell convection (*red/yellow/green*) enclosed between two stable layers (*blue*) in the helium core during core helium flash

of hydrodynamic simulations which are based on the solution of the Navier-Stokes equations and are essentially parameter free. This approach is computationally demanding, and in our case does not yet predict any observables. Nevertheless, our simulations provide some important insight into intrinsically multidimensional processes connected to core shell convection, like convective overshooting, and mixing across entropy barriers. In particular, they revealed a potentially very important, new mixing process operative below the base of such convection zones.

2 Shell Convection in Stellar Cores

Shell convection in stellar cores occurs in a sandwich-like structure where a dynamically unstable zone sustained by nuclear burning is enclosed by two stable regions above and below (Fig. 1). Besides mixing of chemical species the convective shell redistributes the energy released by nuclear burning and keeps the stellar core in quasi-hydrostatic equilibrium. Such convection typically occurs for instance during:

1. Core helium flashes in low-mass red giants (Mocák et al. 2008, 2009, 2010)
2. Core carbon flashes of "super-AGB" stars (Siess et al. 2002)
3. Shell nuclear burning in massive stars (Meakin and Arnett 2007)
4. Thermal pulses of AGB stars (Herwig 2005)

We simulated the first three cases mostly in two but also in three spatial dimensions (2D and 3D simulations, respectively) using initial core structures of metal-rich $1.25\,M_{\odot}$ (Mocák et al. 2008), $9.3\,M_{\odot}$ star (Siess 2006), and $23\,M_{\odot}$ (Meakin and Arnett 2007) stars. None of these simulations led to a violent hydrodynamic event. Initially convection occurred in regions which are unstable according to the Schwarzschild criterion. The convective velocities found in our 3D simulations are close to those predicted by mixing-length theory (MLT). However, our simulations showed an additional non-radial instability at the boundaries of the convection zone, which allowed convection to penetrate inexorably into the adjacent stable layers, thereby increasing the width of the convection zone on a dynamical timescale. Consequently, core convection driven by nuclear burning covers likely larger regions than predicted by MLT theory.

3 Turbulent Entrainment and Entropy Gradients

Convection is able to generate mixing in neighboring stable layers by convective plumes, which can move into these regions due to their momentum. This process is typically called overshooting and refers to localized events. However, if the frequency of these events is high, entropy can change in the affected stable layers. Hence, we can speak of penetration (Brummell et al. 2002). This occurs due to extensive heat exchange between the penetrating convective plumes and the stable layers. Actually, processes at the edges of convection zones appear to be a combination of both overshooting and penetration. We prefer to call them turbulent entrainment (Fernando 1991) which is well described by the divergence of the buoyancy flux at convection boundaries (Meakin and Arnett 2007). We find turbulent entrainment to operate at convection boundaries in all our hydrodynamic models. We thus conclude that convection zones grow in size during dynamic nuclear flashes (nuclear burning in semi-degenerate gas) when entropy gradients given by canonical 1D stellar calculations cannot withstand turbulent entrainment of stable layers. A direct implication of this result for the core helium flash is the occurrence of a hydrogen injection flash, because the entrainment rates found for the convection zone driven by helium burning in our models are of the order of meters per second. Thus, the upper boundary of the convective shell would reach the overlying hydrogen shell within weeks. This is not predicted for solar metallicity stars (Campbell and Lattanzio 2008) as our initial model. The implications of enlarged convection zones for the core carbon flash in intermediate-mass stars remain unexplored, and the case of the oxygen burning shell in massive stars was studied by Meakin and Arnett (2006, 2007).

4 Hydrogen Injection Flash and Mixing Across an Entropy Barrier

Hydrogen injection during the core helium flash is predicted by canonical 1D stellar evolution calculations only for Pop III and extremely metal-poor stars with intrinsic metallicities [Fe/H] < -4]. A similar hydrogen injection phase also occurs:

- At the beginning of the thermally pulsing AGB phase of metal-poor intermediate-mass stars (Chieffi et al. 2001; Siess et al. 2002; Iwamoto et al. 2004)

- In “Late Hot Flasher” stars experiencing strong mass loss on the RGB (Brown et al. 2001; Cassisi et al. 2003)

We refer to these events as “dual flashes” (Campbell and Lattanzio 2008), since they all experience simultaneous hydrogen and helium flashes. These events often lead to a “splitting” of the initial convection zone driven by helium burning due to dredge-down of material from the above-lying hydrogen-rich envelope and rapid CNO burning.

In order to study this episode by means of hydrodynamic simulations, we designed a special stellar model from the helium core of a metal-rich $1.25 M_{\odot}$, where we shifted the hydrogen-rich layers down to the upper edge of the already present helium-burning convection zone. This immediately causes hydrogen to be dredged-down to the hotter layers of the underlying helium-burning convection zone. The results of this first 3D simulation of a dual core flash being driven by hydrogen injection into hot layers of helium-burning convection zone (Fig. 2) are the following:

- Dredge-down of protons across an entropy barrier between the helium-rich layers and the hydrogen shell
- Subtle retreat of the helium-burning convection zone (CVZ-1 in Fig. 2) to smaller radii, and the appearance of a hydrogen-burning convection zone (CVZ-2 in Fig. 2) just above it
- Non-existence of stable radiative layer preventing mixing between the two convection zones, which is in contradiction to 1D stellar evolutionary calculations
- Appearance of the 2nd temperature peak at the base of the hydrogen-burning convection zone, in agreement with 1D stellar evolutionary calculations

We think these results should be qualitatively similar to all dual flash events and to thermal pulses occurring during AGB phase of stars. Hydrodynamic simulations of a thermal pulse with hydrogen mixing into the helium-burning convection zone has already led to improvements in our understanding of Sakurai’s object (Herwig et al. 2011).

5 A New Dynamic Mixing Process Below the Base of Shell Convection Zones

In our 2D and 3D hydrodynamic models of the core helium and carbon flash, we discovered unreported dynamic mixing process operating at the base of convection zones. The mixing manifests itself by cold and over-dense blobs sinking in the direction of gravity, leaving traces of material with higher mean molecular weight μ (Fig. 3). Blobs originate from a layer situated just below the lower edge of the convection zone. We do not see any mixing in our oxygen burning shell models. The driving mechanism of the mixing remains unclear. Possible explanations for the appearance of the sinking cold and dense blobs range from:

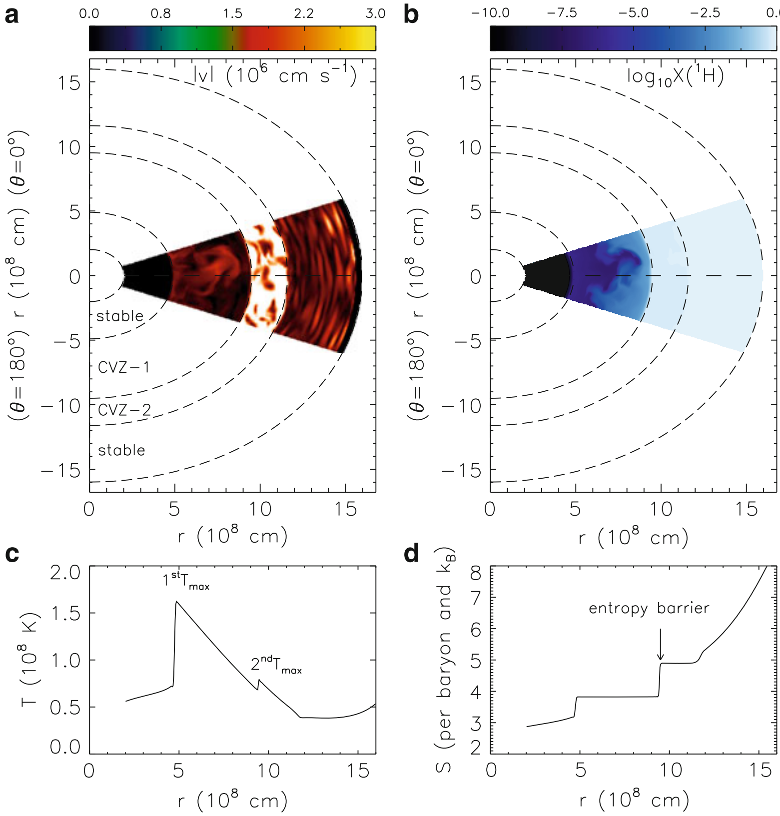

Fig. 2 (**a**) Velocity amplitude |v| (in 10^6 $\mathrm{cm\,s^{-1}}$) and (**b**) hydrogen mass fraction $X(^1H)$ at the onset of the hydrogen injection flash in a meridional plane of a 3D simulation. Panels (**c**) and (**d**) show the angle-averaged radial distributions of temperature and entropy, respectively, at the same time. The two temperature maxima are denoted as $1^{st}T_{max}$ and $2^{nd}T_{max}$, and the entropy barrier is marked by a vertical arrow. Dashed lines enclose distinguished layers, where "CVZ-1" is the convection zone driven by helium burning, "CVZ-2" the convection zone driven by hydrogen burning, and "stable" denotes dynamically stable layers

- The presence of a steep negative mean molecular gradient $\nabla_\mu < 0$ with $\nabla_\mu \equiv d\ln\mu/d\ln P$ (*i.e.,* the molecular weight μ decreases in direction of gravity) destabilizing the layers where the mixing starts
- A strong shear creating peculiar turbulence at the convection boundary due to trapped gravity waves excited by convection in the zone above and indicated by a large value of the square of the Brunt-Väisälä frequency N^2
- A weak dynamic stability of the layers below the convection zone indicated by a relatively small positive value of N^2, which is larger in case of the oxygen burning shell model, and hence mixing is effectively suppressed

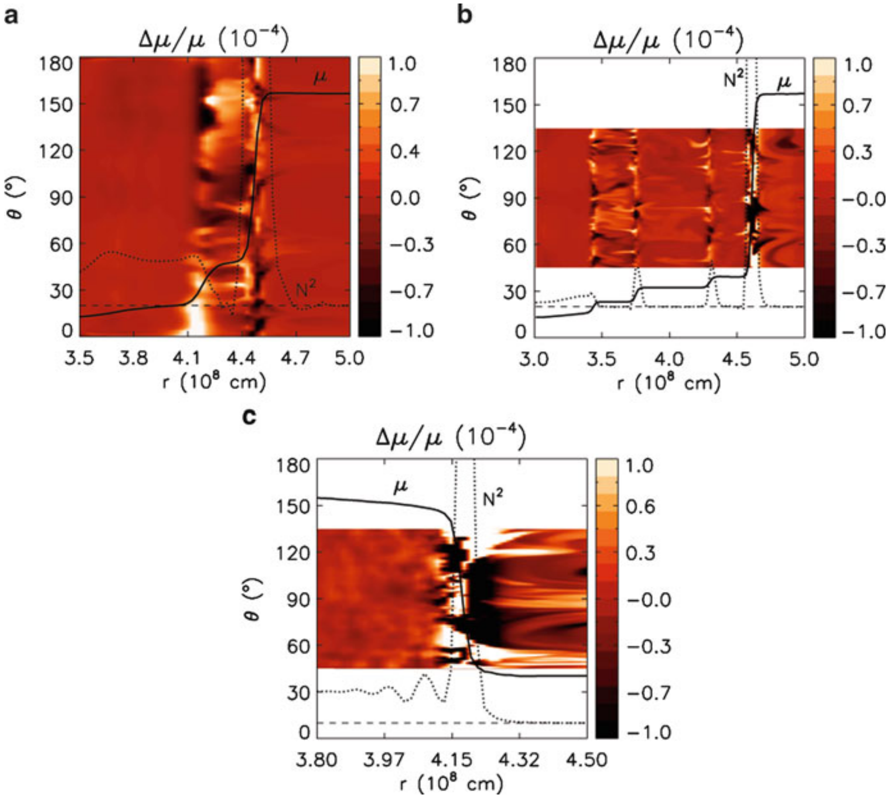

Fig. 3 Relative fluctuations of the mean molecular weight defined as $\Delta\mu/\mu = (\mu - \langle\mu\rangle_\theta) / \langle\mu\rangle_\theta$ (taken from 2D simulations) below the base of shell convection zones during the (**a**) core helium flash, (**b**) core carbon flash, and (**c**) oxygen burning shell shown together with the radial distributions of the mean molecular weight μ (*solid*) and the square of the Brunt-Väisälä frequency N^2. The *horizontal dashed line* corresponds to $N^2 = 0$, and $\langle\rangle_\theta$ denotes the horizontal average at a given radius

- A "turbulence" imitating heat transfer or grid scale diffusion removing heat from patches of gas at the convection zone boundary, hence making some blobs colder and heavy

6 Summary

We investigated convection driven by shell nuclear burning during core helium flash in a low-mass star, core carbon flash in an intermediate-mass star, and in the oxygen burning shell in a massive star using data from 2D and 3D hydrodynamic simulations. We find that all convection zones are growing on a dynamic timescale due to turbulent entrainment. This either implies that the convection zones become enlarged or that the physical conditions at the convection zone boundaries inherited from the initial 1D stellar evolutionary models are inconsistent with real conditions in stars. The enlargement of the helium-burning shell convection zone during the core helium flash may lead to a dual core flash by injection of hydrogen into deeper hot layers of the helium core. With our specifically designed initial model for a hydrogen injection flash we show that such an event is possible and the entropy barrier does not prevent mixing of material from the hydrogen shell into the underlying hot helium-burning convection zone. These findings imply that the stabilizing entropy gradients are less important in stars during flash events. In the hydrogen injection flash simulation, we observe the appearance of a second hydrogen-burning shell residing above the already existing helium-burning convection zone. Radiative stable layers separating the two convection shells are not observed. We think, that these finding should be generally valid for all dual flashes. We also found a new dynamic mixing process below the base of shell convection zones manifesting itself by cold and dense blobs sinking in direction of gravity creating eventually finger-like structures. The origin of this mixing is still under investigation.

Acknowledgements The simulations were performed at the Rechenzentrum Garching of the Max Planck Society. The authors want to thank Casey Meakin for providing us with an initial model of the oxygen burning shell and valuable discussions. Miroslav Mocák acknowledges financial support from the Communauté française de Belgique – Actions de Recherche Concertées.

References

Brown, T.M., Sweigart, A.V., Lanz, T., Landsman, W.B., Hubeny, I.: ApJ **562**, 368 (2001)
Brummell, N.H., Clune, T.L., Toomre, J.: ApJ **570**, 825 (2002)
Campbell, S.W., Lattanzio, J.C.: A&A **490**, 769 (2008)
Cassisi, S., Schlattl, H., Salaris, M., Weiss, A.: ApJ **582**, L43 (2003)
Chieffi, A., Domínguez, I., Limongi, M., Straniero, O.: ApJ **554**, 1159 (2001)
Fernando, H.J.S.: Ann. Rev. Fluid Mech. **23**, 455 (1991)
Herwig, F.: ARA&A **43**, 435 (2005)
Herwig, F., Pignatari, M., Woodward, P.R., et al.: ArXiv e-prints (2011)
Iwamoto, N., Kajino, T., Mathews, G.J., Fujimoto, M.Y., Aoki, W.: ApJ **602**, 377 (2004)
Meakin, C.A., Arnett, D.: ApJ **637**, L53 (2006)
Meakin, C.A., Arnett, D.: ApJ **667**, 448 (2007)
Mocák, M., Müller, E., Weiss, A., Kifonidis, K.: A&A **490**, 265 (2008)
Mocák, M., Müller, E., Weiss, A., Kifonidis, K.: A&A **501**, 659 (2009)
Mocák, M., Campbell, S.W., Müller, E., Kifonidis, K.: A&A **520**, A114+ (2010)
Montalbán, J., D'Antona, F., Kupka, F., Heiter, U.: A&A **416**, 1081 (2004)
Siess, L.: A&A **448**, 717 (2006)
Siess, L., Livio, M., Lattanzio, J.: ApJ **570**, 329 (2002)
Weiss, A., Hillebrandt, W., Thomas, H.-C., Ritter, H.: Cox and Giuli's Principles of Stellar Structure. Gardners Books, East Sussex (2004)