DYNAMICAL PROCESSES IN STELLAR RADIATION ZONES: SECULAR MAGNETOHYDRODYNAMICS OF ROTATING STARS

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ABSTRACT

With the iminent launching of COROT and the preparation of new helioseismology instruments such as GOLF-NG (cf. DynaMICS project), we need a coherent picture of the evolution of rotating stars from their birth to their death. We describe here the modelling of the macroscopic transport of angular momentum and matter in stellar interiors that we have undertaken to achieve this goal. First, we recall in details the dynamical processes that are driving these mechanisms in rotating stars and the theoretical advances we have done. Then, we present our new results of numerical simulations which allow us to follow in 2D the secular hydrodynamics of rotating stars, assuming that anisotropic turbulence enforces a shellular rotation law. Finally, we show how this work is leading to a dynamical vision of the Hertzsprung-Russel diagram in support of asteroseismology and helioseismology and we discuss the different processes that should be studied in next future to improve our description of stellar radiation zones.

Key words: MHD, turbulence, waves, Sun, stars.

1. DYNAMICS OF STELLAR RADIATION ZONES AND DIFFERENTIAL ROTATION

Rotation, and more precisely differential rotation has a major impact on the internal dynamics of stars.
First, as it is known from the theory of rotating stars, rotation induces some-large scale circulations, both in radiation and convection zones, which act to transport simultaneously angular momentum, chemicals but also magnetic field by advection. In radiation zones, the large-scale circulation, which is called the meridional circulation, is due to the differential rotation, to the transport of angular momentum and to the action of perturbing forces, namely the centrifugal force and the Lorentz force (cf. [4], [39], [13], [10], [26]).
Next, differential rotation induces hydrodynamical turbulence in radiative regions through various instabilities: the secular and the dynamical shear instabilities, the baroclinic and the multidiffusive instabilities. In the same way that atmospheric turbulence in terrestrial atmosphere, it acts, to reduce gradients of angular velocity and of chemical composition; thus, it is modelled as a diffusive process (cf. [31], [8], [15]).
Then, rotation has a strong impact on stellar magnetism. For example, it interacts with turbulent convection in convective envelopes of solar-type stars to lead to a dynamo mechanism and, as it is expected from observations, to a cyclic magnetism. In radiation regions, it interacts with fossil magnetic fields where the secular torque exerted by the Lorentz force and the turbulence induced by magnetohydrodynamical instabilities (Tayler-Spruit instability, multidiffusive magnetic instability) have a strong impact on transport of angular momentum and of chemicals (cf. [6], [9], [28], [29], [22], [16], [1], [2]).
Rotation have also strong interactions with waves. Internal waves which are excited at the borders with convective zones, propagate inside radiation zones where they extract or deposit angular momentum where they are damped leading to a modification of the angular velocity profile and of the chemicals distribution (cf. [32], [33], [34], [35], [27], [11]).
Finally, in closed binary systems, where the companion could be star as well as planet, there are transferts of angular momentum between the star, its companion and the orbit due to the dissipation acting on flows induced by the tidal potential, namely the equilibrium tide (cf. [37]) due to the hydrostatic adjustement of the star and the dynamical tide which is due to the excitation of internal waves (cf. [38]). This dynamical evolution modifies the internal rotation of each component that have consequences on the properties of their internal transport.
Note also that rotation modifies stellar winds and mass losses (cf. [14]).
To conclude all the processes with wich rotation interacts transport angular momentum and matter that modifies the

internal angular velocity, the chemical composition and the nucleosynthesis. Therefore, rotation (differential rotation) has imperatively to be taken into account to get a coherent picture of the internal dynamics and the evolution of the stars.

2. THEORETICAL CONTRIBUTIONS

First, the rotational transport of type I where angular momentum and chemicals are transported by the meridional circulation and by the hydrodynamical turbulence due to shear instabilities has been studied. We generalize its present modelling to treat simultaneously the bulk of radiation zones and their interfaces with convective zones, the tachoclines (cf. [17]). Then, we have derived a new prescription for the horizontal turbulent transport which is derived from Couette-Taylor laboratory experiments that allow to study turbulence induced by differential rotation (cf. [18]). However, the introduction of these two hydrodynamical mechanisms in stellar models leads to results which do not agree with observations of solar-type stars, because these have been slowed down by the wind during their evolution and hence the rotational processes are less efficient. Therefore, we consider the rotational transport of type II where chemicals are still transported by meridional circulation and turbulence, but where angular momentum is carried by another process, the two candidates being magnetic field and internal waves. First, we introduce the effect of a fossil magnetic field in a consistent way where we take into account the action of turbulence, differential rotation and meridional circulation on the field but also its feed-back on momentum and heat transports (cf. [19]). Then, we introduce in the modelling of internal waves the effect of the Coriolis force (cf. [20]), that allows to include the gravito-inertial waves in the description of the angular momentum transport. Finally, a coherent treatment of tidal processes has been derived (cf. [21]).

3. MODELLING

3.1. Transport equations

To get a coherent dynamical description of stellar radiation zones, the complete equations of magnetohydrodynamics have to be solved.
The first one is the equation of induction:

$$\partial_t \vec{B} = \underbrace{\vec{\nabla}\wedge\left(\vec{V}\wedge\vec{B}\right)}_{\mathrm{I}} \underbrace{-\vec{\nabla}\wedge\left(||\eta||\otimes\vec{\nabla}\wedge\vec{B}\right)}_{\mathrm{II}}. \qquad (1)$$

It allows to study the temporal evolution of the magnetic field, $\vec{B}$, under its advection by the macroscopic velocity field, $\vec{V}$, (term I) and its ohmic diffusion (term II), $||\eta||$ being the magnetic eddy-diffusivitiy tensor which could be anisotropic due to the stratification of stellar radiation zones. t is the classical time.
The second one is the well-known Navier-Stockes equation, in other words the equation of dynamics:

$$\rho\left[\partial_t\vec{V}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}\right] = -\vec{\nabla}P-\rho\vec{\nabla}\phi + \vec{\nabla}\cdot||\tau|| + \left[\frac{1}{\mu_0}\left(\vec{\nabla}\wedge\vec{B}\right)\right]\wedge\vec{B}. \qquad (2)$$

ρ, P, ϕ are respectively the density, the pressure and the gravitational potential; $||\tau||$ is the Reynolds stress tensor. This equation allows to follow the dynamics of the stellar plasma under the action of the advection $\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}$, the pressure gradient, the gravitational potential, the viscous friction and the Lorentz force. It as to be solved with the continuity equation:

$$\partial_t\rho+\vec{\nabla}\cdot\left(\rho\vec{V}\right)=0. \qquad (3)$$

The last fundamental equation that has to be solved is the equation for the transport of the macroscopic entopy, S:

$$\rho T\left[\partial_t S+\vec{V}\cdot\vec{\nabla}S\right] = \underbrace{\vec{\nabla}\cdot\left(\chi\vec{\nabla}T\right)}_{\mathrm{I}} + \underbrace{\rho\epsilon}_{\mathrm{II}} - \vec{\nabla}\cdot\vec{F}+\mathcal{J}. \qquad (4)$$

This equation describes the transport of entropy by advection with taking into account the thermal diffusion (term I; T and χ are respectively the temperature and the thermal conductivity), the heating due to nuclear reactions (term II; ϵ is the nuclear energy production rate per unit mass), the one due to turbulence:

$$\vec{\nabla}\cdot\vec{F} = -\vec{\nabla}\cdot\left[\rho T||D||\otimes\vec{\nabla}S\right], \qquad (5)$$

where $||D||$ is the eddy-diffusivity tensor, and the ohmic heating:

$$\mathcal{J} = \frac{1}{\mu_0}\left[||\eta||\otimes\left(\vec{\nabla}\wedge\vec{B}\right)\right]\cdot\left(\vec{\nabla}\wedge\vec{B}\right). \qquad (6)$$

μ_0 is the vacuum magnetic permeability.
Finally, the equation for the transport of chemicals:

$$\rho\left[\partial_t c_i+\left(\vec{V}\cdot\vec{\nabla}\right)c_i\right] = \vec{\nabla}\cdot\left(\rho||D||\otimes\vec{\nabla}c_i\right) \qquad (7)$$

has to be solved to study the mixing; c_i is the concentration of the i^{th} chemical which is considered.

3.2. Main assumptions

A multi-scales problem in time and space

Here, secular magnetohydrodynamics and its consequences on stellar evolution is studied. Thus, secular time-scales associated to the nuclear evolution of stars are chosen. Moreover, low angular resolution (expansion in few spherical harmonics) is considered due to the turbulent transport in radiation zones (cf. next paragraph).

Physical processes which have dynamical time-scales and need a high angular resolution description, such as hydro or magnetohydrodynamical instabilities and turbulence, are treated using prescriptions. This is the first step to achieve the highly multi-scales problem of dynamical stellar evolution that could not be yet studied with Direct Numerical Simulation.

Scalar fields (rotation, temperature, chemical concentrations)

Stellar radiation zones are stably stratified regions. Thus, the buoyancy force, which is the restoring force, acts to inhibit turbulent motions in the vertical direction. This leads to a strongly anisotropic turbulent transport where that in the horizontal direction (on an isobar) is more efficient than that in the vertical one. Therefore, horizontal eddy-transport coefficients are larger than those in the vertical direction and the horizontal gradients of scalar fields such as rotation, temperature and chemical concentration are smaller than their vertical gradients. An horizontal expansion in few spherical harmonics is thus allowed, and we get:

$$X\left(r,\theta,\varphi\right)=\overline{X}\left(r,t\right)+\delta X\left(r,\theta,\varphi,t\right)$$
$$\text{with } \delta X\left(r,\theta,\varphi,t\right)=\sum_{l>0}\sum_{m=-l}^{l}\widetilde{X}_m^l\left(r,t\right)Y_l^m\left(\theta,\varphi\right)$$
$$\text{and}\quad \overline{X}\left(r,t\right)\gg\widetilde{X}_m^l\left(r,t\right) \tag{8}$$

where $\overline{X}$ and δX are respectively the horizontal average on an isobar and the fluctuation. r,θ,φ are the classical spherical coordinates.

Vectorial fields

Dynamical equations such as the induction equation or the Navier-Stockes equation are three-dimensional vectorial equations. Here, aim is to study secular magnetohydrodynamics of rotating stars using stellar evolution codes to study the consequences of transport on stellar structure and evolution, these codes being mostly unidimensional. To couple them with transport equations and achieve our goal, we thus proceed as in stellar oscillations theory, expanding vector fields such as macroscopic velocities or magnetic field in vectorial spherical harmonics (see [25]):

$$\vec{u}\left(r,\theta,\varphi,t\right)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\Big\{u_m^l\left(r,t\right)\vec{R}_l^m\left(\theta,\varphi\right)+v_m^l\left(r,t\right)\vec{S}_l^m\left(\theta,\varphi\right)+w_m^l\left(r,t\right)\vec{T}_l^m\left(\theta,\varphi\right)\Big\}. \tag{9}$$

The $\vec{R}_l^m\left(\theta,\varphi\right)$, $\vec{S}_l^m\left(\theta,\varphi\right)$ and $\vec{T}_l^m\left(\theta,\varphi\right)$ are defined as:

$$\vec{R}_l^m\left(\theta,\varphi\right)=Y_l^m\left(\theta,\varphi\right)\widehat{e}_r,\ \vec{S}_l^m\left(\theta,\varphi\right)=\vec{\nabla}_{\mathcal{S}}Y_l^m\left(\theta,\varphi\right)$$
$$\text{and}\quad \vec{T}_l^m\left(\theta,\varphi\right)=\vec{\nabla}_{\mathcal{S}}\wedge\vec{R}_l^m\left(\theta,\varphi\right) \tag{10}$$

where $\vec{\nabla}_{\mathcal{S}}=\widehat{e}_\theta\partial_\theta+\widehat{e}_\varphi\frac{1}{\sin\theta}\partial_\varphi$.

Those expansions in spherical functions of respectively scalar and vectorial fields allow to separate variables in transport equations. Thus, modal equations in r and t only which could be implemented directly in stellar evolution codes are obtained.

3.3. Preliminary definitions

The macroscopic velocity field is expanded as:

$$\vec{V}=r\sin\theta\Omega\left(r,\theta\right)\widehat{e}_\varphi+\dot{r}\widehat{e}_r+\vec{\mathcal{U}}_M\left(r,\theta\right)+\vec{u}\left(r,\theta,\varphi,t\right). \tag{11}$$

The first term, where $\Omega\left(r,\theta\right)$ is the internal angular velocity and $\widehat{e}_\varphi$ is the azimuthal unit vector, is the azimuthal velocity field associated to the differential rotation. Next, the second term II corresponds to the radial lagrangian velocity due to the contractions and dilatations of the star during its evolution, $\widehat{e}_r$ being the radial unit vector. The third term $\vec{\mathcal{U}}_M\left(r,\theta\right)$ is the meridional circulation velocity field which has been presented before and which is due to the differential rotation and the transport of angular momentum. Following the general method concerning the expansion of vector fields, it is expanded in vectorial spherical harmonics:

$$\vec{\mathcal{U}}_M=\sum_{l>0}\left\{U_l\left(r\right)P_l\left(\cos\theta\right)\widehat{e}_r+V_l\left(r\right)\frac{\mathrm{d}P_l\left(\cos\theta\right)}{\mathrm{d}\theta}\widehat{e}_\theta\right\}. \tag{12}$$

The anelastic approximation is adopted, thus filtering out sonic waves, that is justified for the slow meridional circulation. Therefore, one have: $\vec{\nabla}\cdot\left(\rho\vec{\mathcal{U}}_M\right)=0$, that leads to the following relation between the othoradial functions V_l and the radial one U_l:

$$V_l\left(r\right)=\frac{1}{l\left(l+1\right)\rho r}\frac{\mathrm{d}\left(\rho r^2U_l\right)}{\mathrm{d}r}. \tag{13}$$

Finally, $\vec{u}\left(r,\theta,\varphi\right)$ is the velocity field of the internal gravity (or gravito-inertial) waves.

Next, the temperature, T, and the mean molecular weight, μ, are respectively expanded as:

$$T\left(r,\theta\right)=\overline{T}\left(r\right)+\delta T\left(r,\theta\right)$$
$$\text{with}\quad \delta T\left(r,\theta\right)=\sum_{l\geq2}\left[\Psi_l\left(r\right)\overline{T}\right]P_l\left(\cos\theta\right), \tag{14}$$

and

$$\mu\left(r,\theta\right)=\overline{\mu}\left(r\right)+\delta\mu\left(r,\theta\right)$$
$$\text{with}\quad \delta\mu\left(r,\theta\right)=\sum_{l\geq2}\left[\Lambda_l\left(r\right)\overline{\mu}\right]P_l\left(\cos\theta\right); \tag{15}$$

$\overline{T}$ and $\overline{\mu}$ are their horizontal averages, δT and $\delta\mu$ being their fluctuations. Finally, Ψ_l and Λ_l are their relative fluctuations.

Finally, the magnetic field is expanded using its divergence-free property (cf. [3]):

$$\vec{B}(r,\theta) = \vec{\nabla} \wedge \vec{\nabla} \wedge \left(\xi_P(r,\theta)\,\widehat{e}_r\right) + \vec{\nabla} \wedge \left(\xi_T(r,\theta)\,\widehat{e}_r\right). \tag{16}$$

ξ_P and ξ_{T} are respectively the poloidal and the toroidal magnetic stream functions which are expanded in spherical harmonics as:

$$\xi_P(r,\theta) = \sum_{l=1}^{\infty} \xi_0^l(r)\, Y_l^0(\theta) \tag{17}$$

and

$$\xi_T(r,\theta) = \sum_{l=1}^{\infty} \chi_0^l(r)\, Y_l^0(\theta). \tag{18}$$

Here, the mean axisymetric magnetic field is considered ($m = 0$). Therefore, the poloidal field, $\vec{B}_P(r,\theta)$, is in the meridional plane while the toroidal one, $\vec{B}_T(r,\theta)$, is purely azimuthal.

The different fields being now well defined, we have to consider the modal transport equations that has to be implemented in stellar evolution codes.

3.4. Transport equations system

First, we get the two advection-diffusion equations, for respectively ξ_P and ξ_T, that are issued from the spectral expansion of the induction equation in the vectorial spherical harmonics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\xi_0^l \underbrace{-r\mathcal{P}_{\mathbf{Ad};l}\left(U_l,\vec{B}\right)}_{\mathrm{Ia}} = \underbrace{\eta_h r \Delta_l\left(\frac{\xi_0^l}{r}\right)}_{\mathrm{IIa}} \tag{19}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\chi_0^l + \partial_r(\dot{r})\,\chi_0^l \underbrace{-\mathcal{T}_{\mathbf{Ad};l}\left(\Omega,U_l,\vec{B}\right)}_{\mathrm{Ib}} = \underbrace{\left[\partial_r\left(\eta_h\partial_r\chi_0^l\right) - \eta_v l\,(l+1)\,\frac{\chi_0^l}{r^2}\right]}_{\mathrm{IIb}}. \tag{20}$$

The terms Ia and Ib, where $\mathcal{P}_{\mathbf{Ad};l}\left(U_l,\vec{B}\right)$ and $\mathcal{T}_{\mathbf{Ad};l}\left(\Omega,U_l,\vec{B}\right)$ are function of the differential rotation, the meridional circulation and the magnetic field, correspond to the advection of $\vec{B}$ by $\vec{\mathcal{U}}_M$ and to the production of its toroidal component through the shear of differential rotation. Terms IIa and IIb correspond to the turbulent ohmic diffusion, where the possibility of an anisotropic turbulent transport is assumed, with η_v and η_h being respectively the eddy-magnetic diffusivity in the vertical direction and in the horizontal one. The transport of each physical quantity is studied from a lagrangian point of view due to the contractions and to the dilatations of the star during its evolution. The time-lagrangian derivative $\mathrm{d}/\mathrm{d}t$ is defined by:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \partial_t + \dot{r}\partial_r. \tag{21}$$

Then, the azimuthal component of Navier Stockes equation leads to the following advection-diffusion equation for the mean rotation rate on a isobar $\overline{\Omega}(r) = \int_0^\pi \Omega(r,\theta)\sin^2\theta \mathrm{d}\theta / \int_0^\pi \sin^3\theta\mathrm{d}\theta$:

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\overline{\Omega}\right)\underbrace{-\frac{1}{5r^2}\partial_r\left(\rho r^4\overline{\Omega}U_2\right)}_{\mathrm{I}} = + \underbrace{\frac{1}{r^2}\partial_r\left(\rho\nu_v r^4\partial_r\overline{\Omega}\right)}_{\mathrm{II}} + \underbrace{\overline{\Gamma}_{\vec{\mathcal{F}}_{\mathcal{L}}}\left(\vec{B}\right)}_{\mathrm{III}} - \underbrace{\frac{1}{r^2}\partial_r\left[\mathcal{F}_J(r)\right]}_{\mathrm{IV}}. \tag{22}$$

Term I corresponds to the advection of angular momentum by the meridional circulation; term II is the diffusive term associated to the action of the shear-induced turbulence where ν_v is the eddy-viscosity in the vertical direction. These two first terms correspond to the rotational transport of type I. Next, term III is associated to the lorentz force torque $\overline{\Gamma}_{\vec{\mathcal{F}}_{\mathcal{L}}}$. Finally, term IV corresponds to the transport by internal waves, $\mathcal{F}_J(r)$ being the associated mean angular momentum flux on an isobar. These two last terms correspond to the rotational transport of type II. The same type of equation is obtained for the differential rotation in latitude (cf. [17], [19]).

Next, due to the long time-scales associated to the meridional circulation, dynamical terms in the meridional components of the Navier-Stockes equation are filtered, keeping only its hydrostatic terms. Taking the curl of the hydrostatic equation, we get the thermal-wind equation:

$$\varphi\Lambda_l - \delta\Psi_l = \frac{r}{\overline{g}}\mathcal{D}_l\left(\Omega,\vec{B}\right), \tag{23}$$

where $\overline{g}$ is the horizontal average of gravity, the explicit form of $\mathcal{D}_l$ in function of Ω and $\vec{B}$ being given in [19]. The more general equation of state is used (cf. [12]):

$$\frac{\mathrm{d}\rho}{\rho} = \alpha\frac{\mathrm{d}P}{P} - \delta\frac{\mathrm{d}T}{T} + \varphi\frac{\mathrm{d}\mu}{\mu} \tag{24}$$

with $\alpha = (\partial\ln\rho/\partial\ln P)_{T,\mu}$, $\delta = -(\partial\ln\rho/\partial\ln T)_{P,\mu}$ and $\varphi = (\partial\ln\rho/\partial\ln\mu)_{P,T}$, μ being the mean molecular weight.

Finally, the equation for the transport of entropy is expanded in spherical functions, and the following equation for the transport of the temperature fluctuation is obtained:

$$C_p\overline{T}\frac{\mathrm{d}\Psi_l}{\mathrm{d}t} + \underbrace{\Phi\frac{\mathrm{d}\ln\overline{\mu}}{\mathrm{d}t}\Lambda_l}_{\mathrm{I}} + \underbrace{\frac{U_l(r)}{H_p}\left(\nabla_{ad} - \nabla\right)}_{\mathrm{II}} = \frac{L(r)}{M(r)}\mathcal{T}_l(r) + \underbrace{\frac{\mathcal{J}_l}{\overline{\rho}}}_{\mathrm{III}}, \tag{25}$$

$\mathcal{T}_l(r)$ being given by:

$$\begin{aligned}\mathcal{T}_l &= 2\left[1-\frac{\overline{f}_{\mathcal{P}}\left(\Omega,\vec{B}\right)}{4\pi G\overline{\rho}}-\frac{(\overline{\epsilon}+\overline{\epsilon}_{\rm grav})}{\epsilon_m}\right]\frac{\widetilde{g}_l\left(\Omega,\vec{B}\right)}{\overline{g}}\\ &+ \frac{\widetilde{f}_{\mathcal{P},l}\left(\Omega,\vec{B}\right)}{4\pi G\overline{\rho}}-\frac{\overline{f}_{\mathcal{P}}\left(\Omega,\vec{B}\right)}{4\pi G\overline{\rho}}\left(-\delta\Psi_l+\varphi\Lambda_l\right)\\ &+ \frac{\rho_m}{\overline{\rho}}\left\{\frac{r}{3}\partial_r X_l\left(r\right)-\frac{l\left(l+1\right)H_T}{3r}\left(1+\frac{D_h}{K}\right)\Psi_l\right\}\\ &+ \frac{(\overline{\epsilon}+\overline{\epsilon}_{grav})}{\epsilon_m}\left\{X_l\left(r\right)+(f_\epsilon\epsilon_T-f_\epsilon\delta+\delta)\Psi_l\right.\\ &+ \left.(f_\epsilon\epsilon_\mu+f_\epsilon\varphi-\varphi)\Lambda_l\right\},\end{aligned}$$

where

$$X_l\left(r\right)=H_T\partial_r\Psi_l-(1-\delta+\chi_T)\Psi_l-(\varphi+\chi_\mu)\Lambda_l. \quad (26)$$

$\nabla=\partial\ln T/\partial\ln P$ is the logarithmic radiative gradient of temperature while $\nabla_{\rm ad}$ is the adiabatic one. L is the luminosity, M the mass, $\overline{T}$ the horizontal average of the temperature and C_p the specific heat at constant pressure. We have also introduced respectively the pressure and the temperature scale-heights $H_P = |{\rm d}r/{\rm d}\ln P|$ and $H_T = \left|{\rm d}r/{\rm d}\ln\overline{T}\right|$, the thermal diffusivity $K=\overline{\chi}/\overline{\rho}C_p$, the horizontal eddy-diffusivity D_h and $f_\epsilon=\overline{\epsilon}/(\overline{\epsilon}+\overline{\epsilon}_{\rm grav})$, with $\overline{\epsilon}$ and $\overline{\epsilon}_{\rm grav}$ being respectively the mean nuclear and gravitational energy release rates. ϵ_μ and χ_μ are the logaritmic derivatives of ϵ and of the radiative conductivity χ with respect to μ: $\epsilon_\mu = \left(\partial\ln\epsilon/\partial\ln\mu\right)_{P,T}$ and $\chi_\mu = \left(\partial\ln\chi/\partial\ln\mu\right)_{P,T}$, their derivatives with respect to T being noted as ϵ_T and χ_T: $\epsilon_T = \left(\partial\ln\epsilon/\partial\ln T\right)_{P,\mu}$ and $\chi_T = \left(\partial\ln\chi/\partial\ln T\right)_{P,\mu}$. Moreover, we have $\epsilon_m = L/M$ and ρ_m is the mean density inside the considered level surface where $\frac{\overline{g}(r)}{4\pi G}=\rho_m\frac{r}{3}$. Terms $\overline{f}_{\mathcal{P}}$, $\widetilde{f}_{\mathcal{P},l}$ and $\widetilde{g}_l$, the fluctuation of gravity on an isobar, are associated to the meridional perturbing force, $\vec{\mathcal{F}}_{\mathcal{P}}$, namely the sum of the centrifugal force and of the meridional Lorentz force, $\vec{\mathcal{F}}_{\mathcal{L},P}$:

$$\vec{\mathcal{F}}_{\mathcal{P}}=\frac{1}{2}\Omega^2\vec{\nabla}\left(r^2\sin^2\theta\right)+\vec{\mathcal{F}}_{\mathcal{L},P}. \quad (27)$$

If we project $\vec{\mathcal{F}}_{\mathcal{P}}$ on the vectorial spherical harmonics, its explicit expansion in function of Ω, ξ_l and χ_l is derived (cf. [19]).

Note that in a medium of varying composition, we have to take into account the entropy of mixing (cf. [13]). In the simplest case, applicable to main-sequence stars, where the stellar plasma can be approximated by a mixture of hydrogen and helium with a fixed abundance of metals, it can be expressed in terms of the mean molecular weight only:

$${\rm d}S=C_p\left[\frac{{\rm d}T}{T}-\nabla_{\rm ad}\frac{{\rm d}P}{P}+\Phi\left(P,T,\mu\right)\frac{{\rm d}\mu}{\mu}\right], \quad (28)$$

where Φ is a function of the metal mass fraction and of μ, the mean molecular weight.

Finally, ohmic heating has been expanded in spherical functions, $\mathcal{J}_l$ being the modal radial functions. Their explicit expression in function of ξ_l and χ_l is given in [19].

Term I, II and III in eq. 25 correspond respectively to the multi-species caracteristic of the stellar plasma, to the advection of temperature fluctuations by the meridional circulation and to the ohmic heating. Next, the first two lines in $\mathcal{T}_l$ constitute what is called the barotropic term of the meridional circulation which is generated by the perturbation of thermal imbalance by $\vec{\mathcal{F}}_{\mathcal{P}}$. The third line correspond to the thermal diffusion. In fact, if we keep only the higher-order derivatives, we obtain $\frac{\rho_m}{\overline{\rho}}\frac{r}{3}\partial_r\left(H_T\partial_r\Psi_l\right)$ which is directly associated with the temperature laplacian. Finally, the last two lines correspond to the energy production by nuclear reactions and to the heating due to the radial adjustements of the star during its evolution.

The last transport equation which is solved is that for the the mixing of chemicals. Expanding it on an isobar, it is obtained for the average of the concentration of each chemical, c_i:

$$\begin{aligned}&\rho\frac{\rm d}{{\rm d}t}\overline{c}_i+\frac{1}{r^2}\partial_r\left[r^2\rho\overline{c}_iU_i^{\rm diff}\right]\\ &= \frac{1}{r^2}\partial_r\left[r^2\rho\left(D_v+D_{\rm eff}\right)\partial_r\overline{c}_i\right], \quad (29)\end{aligned}$$

where $U_i^{\rm diff}$ is the velocity associated to the microscopic diffusion processes while the strong horizontal turbulence leads to the erosion of the advective transport of chemicals which becomes a diffusive process (cf. [5]) with the following diffusion coefficient:

$$D_{\rm eff}=\sum_{l>0}\frac{\left(rU_l\right)^2}{l\left(l+1\right)\left(2l+1\right)D_h}. \quad (30)$$

Then, taking the definition of the mean molecular weight: $\frac{1}{\mu}=\sum_i\left[\left(1+Z_i\right)/A_i\right]c_i$ (A_i and Z_i are respectively the number of nucleons and of protons of the i$^{\rm th}$ element which is considered), the following advection-diffusion equation is obtained for Λ_l:

$$\frac{{\rm d}\Lambda_l}{{\rm d}t}-\frac{{\rm d}\ln\overline{\mu}}{{\rm d}t}\Lambda_l-\frac{U_l}{H_p}\nabla_\mu=-\frac{l\left(l+1\right)}{r^2}D_h\Lambda_l \quad (31)$$

where $\nabla_\mu=\frac{\partial\ln\overline{\mu}}{\partial\ln P}$.

4. NUMERICAL SIMULATION OF SECULAR TRANSPORT: THE HYDRODYNAMICAL CASE WITH A 'SHELLULAR' ROTATION

4.1. Hydrodynamical transport equations system

The numerical simulations presented here were computed with the dynamical stellar evolution code STAREVOL and the reader is referred to [30], [23], [24] for a detailed description. In the hydrodynamical case ($\vec{B} = \vec{0}$) where we assume that the differential rotation is shellular $\Omega(r,\theta) = \overline{\Omega}(r)$ due to the stronger horizontal turbulence which enforce the angular velocity to be constant on an isobar, the system is reduced to:

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\overline{\Omega}\right) - \frac{1}{5r^2}\partial_r\left(\rho r^4 \overline{\Omega} U_2\right) = \frac{1}{r^2}\partial_r\left(\rho\nu_v r^4 \partial_r \overline{\Omega}\right) \tag{32}$$

and to the $l = 2$ mode of Eqs. 23,25 and 31 (here internal waves are not taken into account). It has been now implemented in STAREVOL.

This fourth-order system (cf. [39] for a more detailed discussion) is solved with the following boundary conditions for $\overline{\Omega}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\int_0^{r_b} r^4\rho\Omega \mathrm{d}r\right] = \frac{1}{5}r^4\rho\overline{\Omega}U_2 + \rho\nu_v r^4\partial_r\overline{\Omega}, \tag{33}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\int_{r_t}^{R} r^4\rho\Omega \mathrm{d}r\right] = -\frac{1}{5}r^4\rho\overline{\Omega}U_2 - \rho\nu_v r^4\partial_r\overline{\Omega} - \mathcal{F}_\Omega \tag{34}$$

and

$$\partial_r\overline{\Omega} = 0 \text{ at } r = r_b \text{ and } r = r_t \tag{35}$$

where r_b and r_t are respectively the radius of the bottom and of the top of the considered radiation zone; $\mathcal{F}_\Omega$ is the flux of angular momentum which is extracted at the surface by the wind. The condition given in Eq. 35 has been chosen using our knowledge of angular velocity inside the solar convection zone, where $\Omega(r,\theta)$ depends mainly on the latitude. However, we are perfectly conscious that in the general case $\partial_r\overline{\Omega}$ has to be matched with its value in the adjacent convection zones that has to be determined by observations or by numerical simulations. Those values could have a strong impact on the mixing which then occurs (cf. [24]).
The two boundary conditions for Eq. 29 are respectively at $r = r_b$ and $r = r_t$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\overline{c}_i\int_0^{r_b} r^2\rho \mathrm{d}r\right] = -r^2\rho\left(U_i^{\mathrm{diff}}\overline{c}_i\right) + r^2\rho\left(D_v + D_{\mathrm{eff}}\right)\partial_r\overline{c}_i \tag{36}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\overline{c}_i\int_{r_t}^{R} r^2\rho \mathrm{d}r\right] = r^2\rho\left(U_i^{\mathrm{diff}}\overline{c}_i\right) - r^2\rho\left(D_v + D_{\mathrm{eff}}\right)\partial_r\overline{c}_i - \overline{\dot{M}}\overline{c}_i \tag{37}$$

where $\overline{\dot{M}}$ is the horizontal average of the mass-loss rate at the surface (one has to recall that $\dot{M}$ is a function of the latitude if the effect of rotation on stellar winds is taken into account (cf. [14])).

4.2. Application to a $1.5M_\odot$ star with a solar metallicity

The results presented here are issued from the numerical simulation of the evolution of a $1.5M_\odot$ star with a solar metallicity ($Z = 0.02$) and an initial equatorial rotation velocity $v_{\mathrm{ini}} = 100$ km.s^{-1}. The age is 7.604×10^8 yr with a central Hydrogen mass fraction $X_c = 0.57$. It is now possible to follow for each time-step the internal hydrodynamics of the radiation zone(s) of the star which is studied, following simultaneously the differential rotation profile (see Fig. 1), the temperature and the mean molecular weight excesses due to differential rotation (see Fig. 2 & 3) and the associated meridional circulation pattern (see Fig. 4).

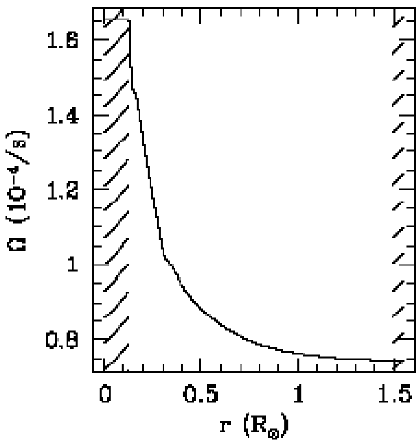

Figure 1. The differential rotation profile.

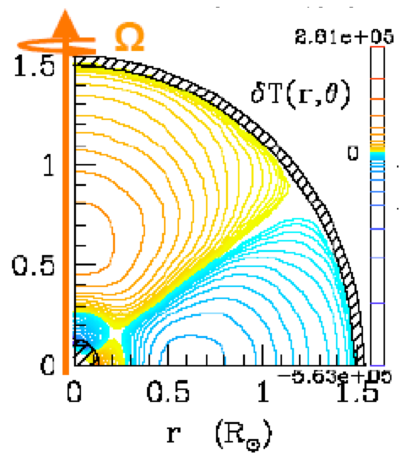

Figure 2. The T-excesses $\overline{T}\Psi_2 P_2(\cos\theta)$. It reachs $+2.81\times10^5K$ in the inner region closer to the polar axis and -5.63×10^5K in the inner equatorial region and become smaller near the surface.

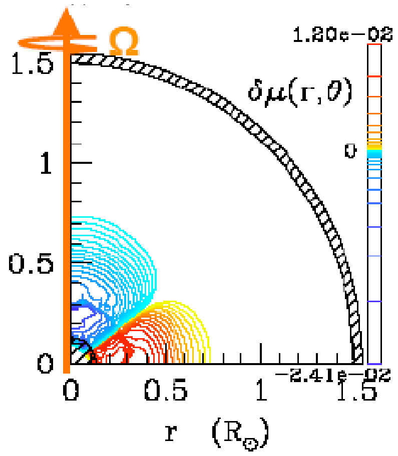

Figure 3. The μ-excesses $\overline{\mu}\Lambda_2 P_2(\cos\theta)$. They only occur close to the core and they are positive near the equatorial plane.

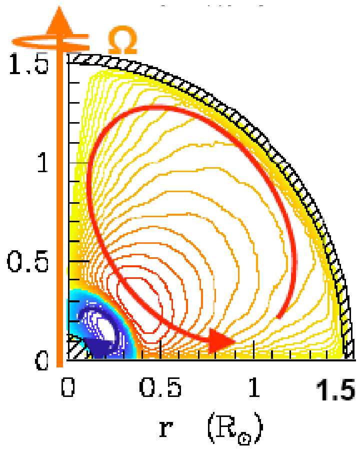

Figure 4. Meridional circulation currents. In this model, the outer cell is turning counterclockwise allowing the equatorial extraction of angular momentum by the wind.

Moreover, diagnosis tools have been developped to identify dominant physical processes in the angular momentum transport, the meridional circulation and the chemicals mixing.

Considering Fig. 5, one can easily identify that here angular momentum transport is dominated by the advection by the meridional circulation, its flux transported by the shear-induced turbulence being smaller at least of an order of magnitude except in the region near the center. Then, looking at Fig.6, one can easily identify that meridional circulation is mainly driven by the barotropic terms and by the thermal diffusion. The term due to the nuclear energy production has to be taken into account only in the region of the star where nuclear reactions occur, here in the center, while the non-stationary term is completely negligible since the star which is studied here is a main-sequence star where structural adjustements are weak. Finally, if transport coefficients are studied, it can be indentified that the meridional circulation is the dominating process in the transport of chemicals while our fundamental hypothesis concerning the turbulent transport is verified ($D_v \ll D_h$).

Work is now in progress to implement differential rotation in latitude and transport by magnetic field and gravito-inertial waves, those two last processes being crucial to explain the internal rotation profile of the Sun and the properties of low-mass stars. This will lead to an hydrodynamical (and then to a MHD) vision of stellar evolution ready for helio and asteroseismic diagnosis.

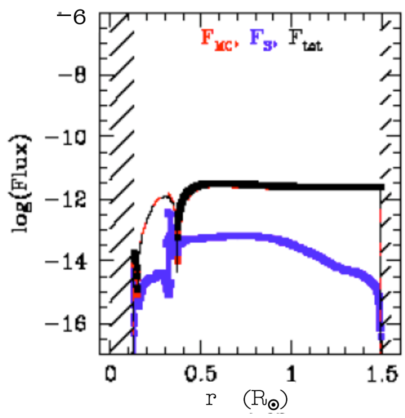

Figure 5. Logarithm of the total flux of angular momentum (black line) and of that transported by meridional circulation, $F_{\rm MC}(r) = \frac{1}{R_\odot^4}\frac{1}{5}\rho r^4 \overline{\Omega} U_2$, (red line) and by shear-induced turbulence, $F_{\rm S}(r) = \frac{1}{R_\odot^4}\rho \nu_v r^4 \partial_r \overline{\Omega}$, (blue line).

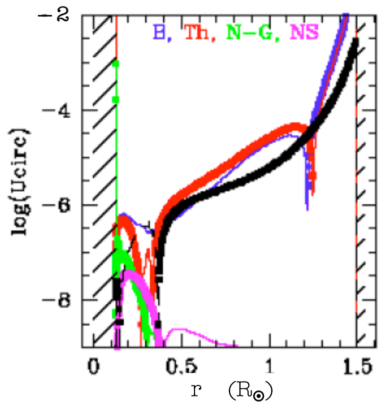

Figure 6. Logarithm of the meridional circulation (black line), the barotropic term (blue line), the thermal diffusion term (red line), the nuclear energy production and heating due to gravitational adjustements term (green line) and the non-stationarity term (purple line) profiles.

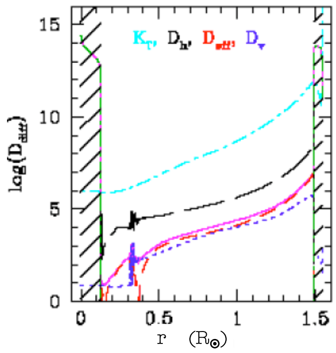

Figure 7. Logarithm of the themal diffusivity (cyan line), the horizontal eddy-diffusivity (black line), the effective diffusivity associated to meridional circulation (red line), the vertical eddy-diffusivity (blue line) and the total vertical diffusivity, $D_t = D_v + D_{\rm eff}$, (purple line) profiles.

5. CONCLUSION

In this work, a coherent description of dynamical transport processes which take place in stellar radiation zones has been undertaken. Each of them and of their respective effects on angular momentum and chemicals transport has been indentify and modelled in a consistent way. For the first time, a two-dimensional picture of internal dynamics of stellar radiation zones is obtained and the first step of the numerical implementation of theoretical results in stellar evolution codes, namely the purely hydrodynamical case where it is assumed that the strong horizontal turbulent transport enforces a shellular rotation law, has been achieved. Moreover, work is in progress to implement differential rotation in latitude, magnetism, gravito-inertial waves and tides while theoretical work is engaged to give prescriptions for MHD instabilities, waves excitation and tides dissipation. Thus, with the actual and forthcoming helioseismology and asteroseismology spatial missions such as SOHO, GOLF-NG (DynaMICS project, cf. [36]), MOST and COROT, with the powerful ground-based instruments such as ESPaDOnS, HARPS and the VLT, with the development numerical simulations and of physics instruments which allow to make laboratory experiments relevant for astrophysical plasmas, we hope to be able in the near future to obtain a dynamical vision of the Hertzsprung-Russel diagram in support of helio and asteroseismology.

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